Name: ____________________________

Math 1121 Exam #3

No calculators! Show all your work for everything.

$$\frac{d}{dx}e^x = e^x \qquad \int e^x\,dx = e^x + C$$

$$\frac{d}{dx}a^x = a^x \ln a \qquad \int a^x\,dx = \frac{1}{\ln a}a^x + C$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C \qquad \int a^{kx}\,dx = \frac{1}{k\ln a}a^{kx} + C$$

Question 1. (14 points) Please give intervals where this function is concave up and concave down:

$$f(x) = x^2(x^2 + x + 2)$$

This problem had a typo I didn't realize until after the test! I'm so sorry for that – I gave full credit to anyone who found the 2nd derivative correctly. Please contact me with any concerns or complaints.

Question 2. (14 points) Please use the second derivative test to find the x-values of the relative extrema of $f(x)$, and say for each one if it is a maximum or a minimum.

$$f(x) = x^4 - 8x^2 + 3$$

$f'(x) = 4x^3 - 16x$
$= 4x(x^2 - 4)$
$= 4x(x+2)(x-2)$

$f''(x) = 12x^2 - 16$
$= 4(3x^2 - 4)$

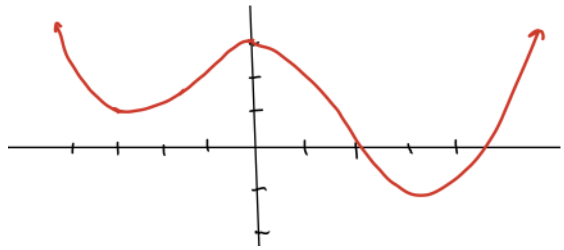
crit #s: $x=0$, $x=2$, $x=-2$

$f''(0) = 4(0-4) = -$ so $x=0$ is a rel max

$f''(2) = 4(3\cdot 2^2 - 4) = +$ so $x=2$ is a rel min

$f''(-2) = 4(3(-2)^2 - 4) = +$ so $x=-2$ is a rel min

Question 3. This whole page is about this picture of $f(x)$:



a) (5 points) Please give intervals where $f(x)$ is increasing and decreasing.

inc : $(-3, 0)$ & $(3, \infty)$

dec: $(-\infty, -3)$ & $(0, 3)$

b) (5 points) Please give intervals where $f(x)$ is concave up and concave down.

up: $(-\infty, -1)$ & $(2, \infty)$

down: $(-1, 2)$

c) (3 points) Please give the x-values of any inflection points, or say if there are none.

$x = -1$, $x = 2$

d) (5 points) Please give the x-values for any relative extrema of $f(x)$, and say for each one if it is a maximum or minimum.

$x = -3$ is rel min

$x = 0$ is rel max

$x = 3$ is rel min

e) (5 points) Please find the x-values for any absolute extrema of $f(x)$ on the interval $[1, 4]$, and say for each one if it is a maximum or minimum.

$x = 1$ is the abs max

$x = 3$ is the abs min

Question 4. (7 points each) Please find these antiderivatives:

a) $\int 7x^4 - 8x^3 - x + 5\,dx$

$$7 \cdot \frac{1}{5}x^5 - 8 \cdot \frac{1}{4}x^4 - \frac{1}{2}x^2 + 5x + C$$

b) $\int 5 \cdot 4^{2x} + \frac{1}{6x^4}\,dx$ $= \int 5 \cdot 4^{2x} + \frac{1}{6}x^{-4}\,dx$

$$5 \cdot \frac{1}{2\ln 4} 4^{2x} + \frac{1}{6} \cdot \frac{1}{-3}x^{-3} + C$$

c) $\int 4x^2(x^2 - \frac{1}{x^3})\,dx$ $= \int 4x^4 - 4x^{-1}\,dx$

$$= 4 \cdot \frac{1}{5}x^5 - 4\ln|x| + C$$

Question 5. (13 points) Please find the absolute extrema of $f(x) = x^3 - 3x^2$ on the interval $[-2, 1]$.

$f'(x) = 3x^2 - 6x$

$= 3x(x-2)$

crit #s: $x=0$, $x=2$

x	y	
0	$0^3 - 3 \cdot 0^2 = 0$	← max
$\cancel{2}$ (outside the interval)		
−2	$(-2)^3 - 3(-2)^2 = -8 - 3 \cdot 4 = -20$	← min
1	$1^3 - 3 \cdot 1^2 = -2$	

$x=0$ is the abs max.

$x=-2$ is the abs min.

Question 6. (15 points) Please sketch the graph of

$$f(x) = \frac{4+2x}{5-x}$$

Show enough work so that I can tell why your picture looks the way it does.

Asymptotes:

Horz: $\lim_{x\to\infty} \frac{4+2x}{5-x} = \frac{2}{-1} = -2$

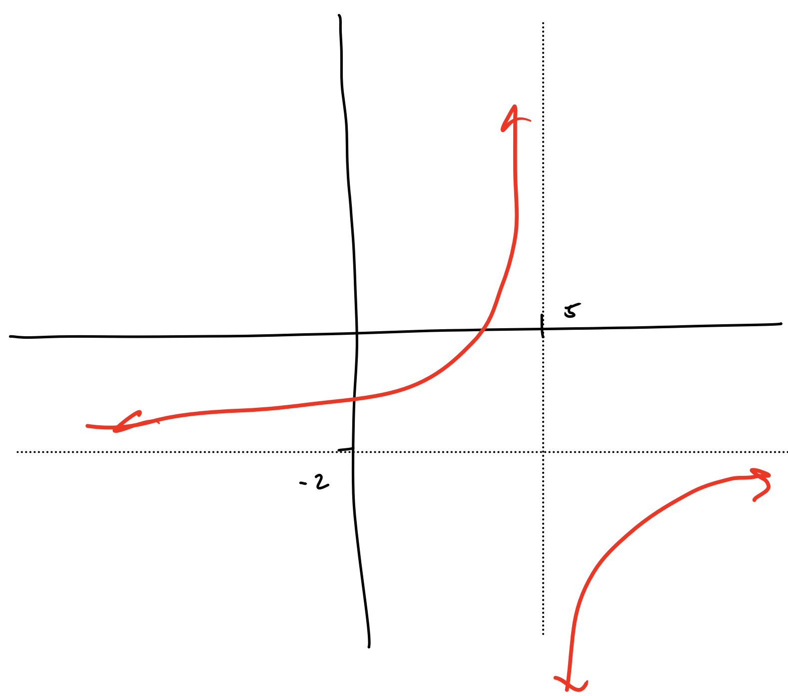
Vert: $5-x=0$

$x=5$

$f'(x) = \frac{(5-x)\cdot 2 - (4+2x)\cdot -1}{(5-x)^2}$

$= \frac{10-2x+4+2x}{(5-x)^2}$

$\boxed{f'(x) = \frac{14}{(5-x)^2}}$



crit #s

$f'=0$: $\frac{14}{(5-x)^2}=0$

$\cancel{14=0}$

f' DNE: $(5-x)^2=0$

$x=5$

inc/dec:

	5	
f': +	DNE	+

$f'(0) = \frac{14}{(\ )^2} = +$

$f'(6) = \frac{14}{(\ )^2} = +$