Name: _____

Math 1171 Exam #1

No calculators! Show all your work for everything. You don't need to simplify your answers unless I say so.

Question 1. (7 points) Please use the definition of the derivative to find the derivative of $f(x) = \frac{1}{2x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{2(x+h)} - \frac{1}{2x}}{h} = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{2(x+h)}\cdot\frac{x}{x} - \frac{1}{2x}\cdot\frac{x+h}{x+h}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x}{2(x+h)x} - \frac{x+h}{2(x+h)x}\right) = \lim_{h\to 0} \frac{1}{h}\cdot\frac{-h}{2x(x+h)}$$

$$= \lim_{h\to 0} \frac{-1}{2x(x+h)} = \frac{-1}{2x^2}$$

Question 2. (7 points) Please find the equation of the line tangent to the curve of $y = \sin x \cos x$ at the point $x = \frac{\pi}{2}$.

Slope: $y' = \sin x \cdot -\sin x + \cos x \cdot \cos x = -\sin^2 x + \cos^2 x$

at $x = \pi/2$: $-\sin^2 \pi/2 + \cos^2 \pi/2 = -1^2 + 0^2 = -1$

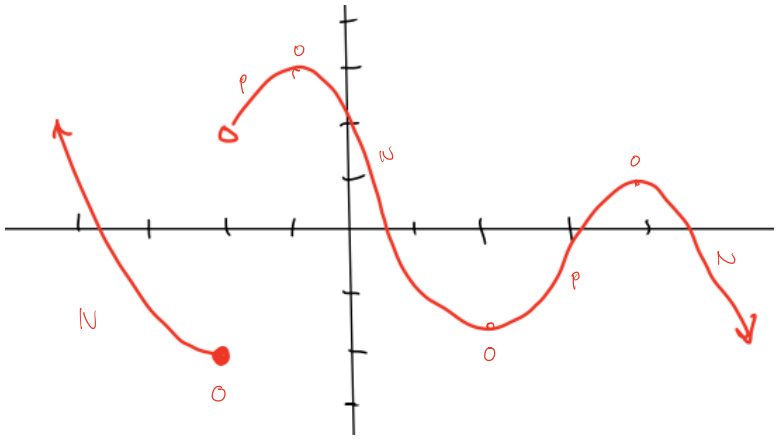
y-value at $x = \pi/2$: $y = \sin \pi/2 \cos \pi/2 = 1 \cdot 0 = 0$

point-slope: $y - y_0 = m(x - x_0)$

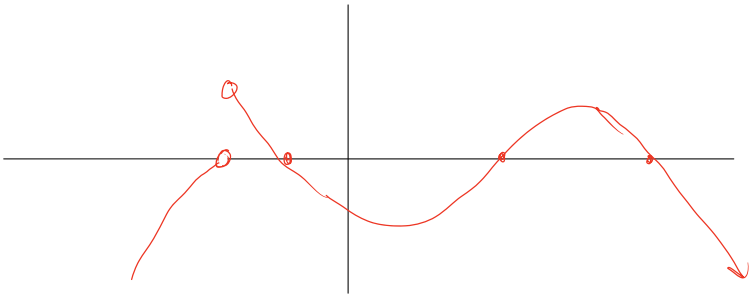
$$y - 0 = -1(x - \pi/2)$$

$$y = -x + \pi/2$$

Question 3. This whole page is about this function $f(x)$:



a) (7 points) Please draw a graph of $f'(x)$. (Hint: where you see a jump in $f(x)$, expect a jump in $f'(x)$.)



b) (3 points) Please give intervals of x values where $f'(x) > 0$.

$(-2,-1)$, $(2,4)$

Please find these: (2 points each)

c) $f(0)$ 2

d) $f(-2)$ -2

e) $\lim_{x\to -2^-} f(x)$ -2

f) $\lim_{x\to -2^+} f(x)$ 2

g) $\lim_{x\to -2} f(x)$ DNE

h) (4 points) Let $g(x) = xf(x)$, and please find $g'(4)$.

$g'(x) = x f'(x) + f(x)$

$g'(4) = 4 f'(4) + f(4)$

$= 4 \cdot 0 + 1 = 1$

Question 4. (7 points each) Please find each limit:

a) $\lim_{x\to 4} \dfrac{x^2-2x-8}{x^2-5x+4}$

$$\lim_{x\to 4} \frac{(x-4)(x+2)}{(x-4)(x-1)} = \lim_{x\to 4} \frac{x+2}{x-1} = \frac{4+2}{4-1} = \frac{6}{3} = 2$$

b) $\lim_{x\to -1} \dfrac{(x+4)^2-9}{4-(x+3)^2}$

$$\lim_{x\to -1} \frac{x^2+8x+16-9}{4-(x^2+6x+9)} = \lim_{x\to -1} \frac{x^2+8x+7}{-x^2-6x-5} = \lim_{x\to -1} \frac{(x+1)(x+7)}{-(x+1)(x+5)}$$

$$= \lim_{x\to -1} \frac{x+7}{-(x+5)} = \frac{-1+7}{-(-1+5)} = \frac{6}{-4} = -3/2$$

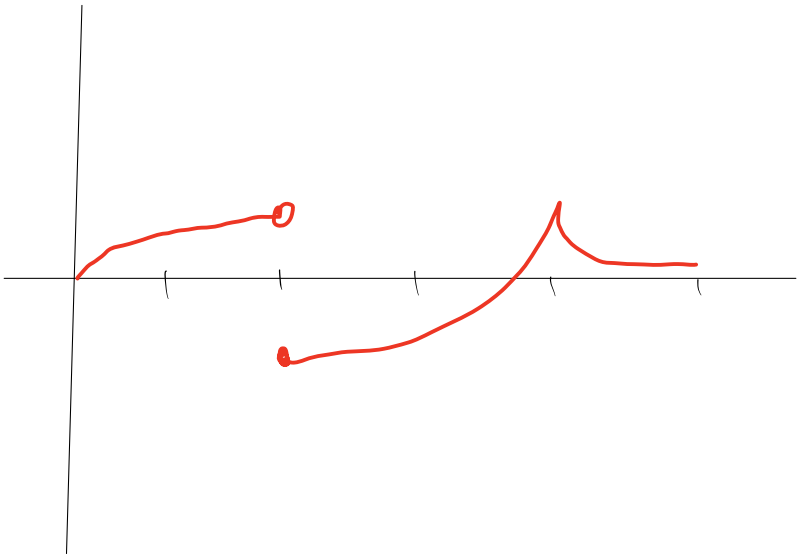
c) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x+2}-2}$ $\cdot \dfrac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$

$$= \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{(\sqrt{x+2})^2 - 2^2} = \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{x+2-4}$$

$$= \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{x-2} = \sqrt{2+2}+2$$

$$= \sqrt{4}+2 = 4$$

Question 5. (5 points) Please draw a picture (invent your own graph) of a function with domain $[0, 5]$ which is continuous everywhere on the domain except $x = 2$, and is differentiable everywhere on the domain except $x = 2$ and $x = 4$.



Question 6. (5 points) Please find any x-values where the slope of the graph is horizontal for:

$$y = x^3 - 6x^2 + 12x + 4$$

$y' = 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)^2$

$y' = 0$ when $3(x-2)^2 = 0$

$(x-2)^2 = 0$

$x - 2 = 0$

$\boxed{x = 2}$

Question 7. (26 points) In each part, please find the derivative:

a) $5x^2 - 7x + \frac{4}{x^3} + \frac{1}{4x^3} - \frac{4}{\sqrt{x}}$ $= 5x^2 - 7x + 4x^{-3} + \frac{1}{4}x^{-3} - 4x^{-1/2}$

$$10x - 7 - 12x^{-4} + \frac{1}{4} \cdot 3x^{-4} + 2x^{-3/2}$$

b) $4x^2(x + \sin x)$

$$4x^2 \cdot (1 + \cos x) + (x + \sin x) \cdot 8x$$

c) $(4x \sin x)^3$

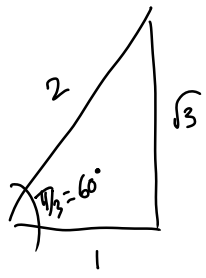
$$3(4x \sin x)^2 \cdot (4x \cdot \cos x + \sin x \cdot 4)$$

d) $\frac{\cos x}{(5x^2 - 7x)^2}$

$$\frac{(5x^2 - 7x)^2 \cdot -\sin x - \cos x \cdot 2(5x^2 - 7x) \cdot (10x - 7)}{(5x^2 - 7x)^4}$$

Question 8. (5 points) For $f(x) = x\sin(\frac{\pi}{6}x)$, please find $f'(2)$. Simplify any trig functions in your answer.

$$f'(x) = x \cdot \cos\left(\frac{\pi}{6}x\right) \cdot \frac{\pi}{6} + \sin\left(\frac{\pi}{6}x\right) \cdot 1$$



So $f'(2) = 2\cos(\pi/3) \cdot \pi/6 + \sin(\pi/3)$

$$= 2 \cdot \frac{1}{2} \cdot \pi/6 + \frac{\sqrt{3}}{2}$$

$$= \pi/6 + \sqrt{3}/2$$