Name: ____________________________

Math 1171 Exam #2

No calculators! Show all your work for everything. You don't need to simplify your answers unless I say so.

Question 1. (10 points) Please give an example of a function $f(x)$ where $\lim_{x\to\infty} f(x) \neq \lim_{x\to-\infty} f(x)$, and show that the two limits are different.

$$f(x) = \frac{\sqrt{x^2+1}}{x}$$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2/x^2 + 1/x^2}}{x/x} = \lim_{x\to\infty} \frac{\sqrt{0+1}}{1} = 1$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x\to-\infty} \frac{-\sqrt{x^2/x^2 + 1/x^2}}{x/x} = \lim_{x\to-\infty} \frac{-\sqrt{0+1}}{1} = -1$$

Question 2. (10 points) Please use a linear approximation to estimate $\sin 3$. (That's 3 radians, not 3 degrees.)

$f(x) = \sin x$ use $a = \pi$, since $\pi \approx 3$.

$f'(x) = \cos x$

$f(a) = \sin\pi = 0$

$f'(a) = \cos\pi = -1$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= 0 + -1(x-\pi)$$
$$= \pi - x$$

so $\sin 3 \approx L(3) = \pi - 3$

$= .1415\ldots$

Question 3. (12 points) Cameraman Dan is filming a rocket launch and has to angle up his camera to follow the rocket as it ascends. If Cameraman Dan is 100m from the launch site, and the rocket is ascending at a constant speed of 5m/s, how quickly is the angle of Cameraman Dan's camera changing when the angle is 60°? Give your answer with units.

$$\tan\theta = \frac{h}{100} \qquad \cos 60^\circ = 1/2$$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{100}\frac{dh}{dt}$$

solve for $\frac{d\theta}{dt}$, plug $\theta = 60^\circ$, $\frac{dh}{dt} = 5$

$$\frac{1}{(\cos 60^\circ)^2} \cdot \frac{d\theta}{dt} = \frac{1}{100} \cdot 5$$

$$\frac{1}{(1/2)^2} \cdot \frac{d\theta}{dt} = \frac{1}{20}$$

$$\frac{d\theta}{dt} = \frac{1}{20} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{80} \ \frac{\text{radians}}{\text{sec}}$$

Question 4. (10 points) Please find the x-values of any local extrema for:

$$f(x) = x^4 - 8x^2 + 3.$$

For each one, say whether it is a maximum or a minimum.

$$f'(x) = 4x^3 - 16x$$
$$= 4x(x^2 - 4)$$
$$= 4x(x-2)(x+2)$$

$\underline{f' = 0}$ $\quad x = 0, \quad x = 2, \quad x = -2$

		-2		0		2	
f':	$-$	0	$+$	0	$-$	0	$+$

$f'(-3) = 4(-3)(-3-2)(-3+2)$
$= + \cdot - \cdot - \cdot - = -$

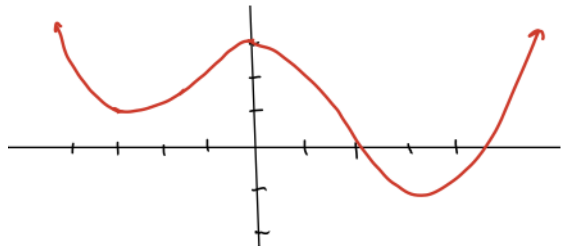
$f'(-1) = 4(-1)(-1-2)(-1+2) = +$
$\quad + \ - \ - \ +$

$f'(1) = 4(1)(1-2)(1+2) = -$
$\quad + \ + \ - \ +$

$f'(3) = 4(3)(3-2)(3+2) = +$

$\boxed{\begin{array}{l} x = -2 \text{ is a local } \underline{\text{min}} \\ x = 0 \text{ is a local } \underline{\text{max}} \\ x = 2 \text{ is a local } \underline{\text{min}} \end{array}}$

Question 5. This whole page is about this function $f(x)$:



a) (4 points) Please say specifically what the Mean Value Theorem tells us about $f(x)$ when considering the interval $[-1, 1]$. (Fully simplify and numbers or formulas in your answer.)

$$\frac{f(1)-f(-1)}{1--1} = \frac{2-2}{2} = 0$$

MVT says there is some c in $[-1,1]$ with $f'(c) = 0$

b) (3 points) Please give intervals where $f(x)$ is increasing and decreasing.

inc: $(-3,0)$ & $(3,\infty)$

dec: $(-\infty,-3)$ & $(0,3)$

c) (3 points) Please give intervals where $f(x)$ is concave up and concave down.

up: $(-\infty, -1)$ & $(2, \infty)$

down: $(-1, 2)$

d) (2 points) Please give the x-values of any inflection points, or say if there are none.

$x = -1, \quad x = 2$

e) (3 points) Please give the x-values for any relative extrema of $f(x)$, and say for each one if it is a maximum or minimum.

$x = -3$ is a local min

$x = 0$ is a local max

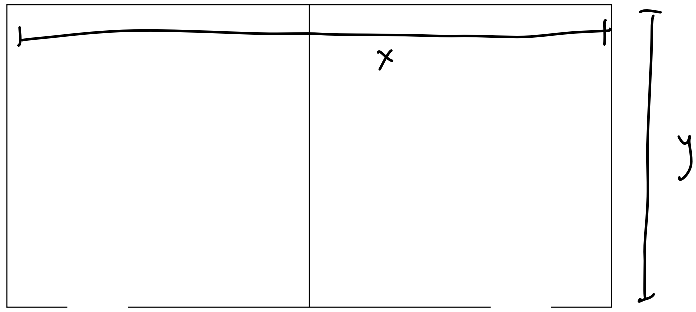
$x = 3$

f) (3 points) Please find the x-values for any absolute extrema of $f(x)$ on the interval $[1, 4]$, and say for each one if it is a maximum or minimum.

abs max at $x = 1$

abs min at $x = 3$

Question 6. (15 points) I am building two adjacent fenced-in areas, each of which has a gap for a driveway, like so:



The gaps are each 5 feet wide. How wide should the enclosure be (all the way across both parts) to maximize the enclosed area, if I can use a total of 80~~m~~ ft of fencing? (You don't need to compute the maximum area, just the width.)

$A = xy$

$3y + 2x - 10 = 80$

$3y + 2x = 90$

$3y = 90 - 2x$

$y = 30 - \frac{2}{3}x$

$A(x) = x\left(30 - \frac{2}{3}x\right)$

$A(x) = 30x - \frac{2}{3}x^2$

interval: smallest $x = 0$

biggest $x = 45$ (when $y = 0$)

$[0, 45]$

crit. #s

$A'(x) = 30 - \frac{4}{3}x$

$30 - \frac{4}{3}x = 0$

$\frac{4}{3}x = 30$

$x = 90/4$ This will be the max since the interval endpts both give $A = 0$.

So the width should be $90/4$ to maximize the area.

Question 7. (15 points) Please sketch the graph of $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 1$. As part of your process, find and plot any critical and inflection points, and use the intervals of concavity and increase & decrease. (You don't need to find symmetries or other stuff unless you want to.)

$f'(x) = x^2 + 2x - 3$

$= (x+3)(x-1)$

$f''(x) = 2x + 2$

$= 2(x+1)$

critical pts: $x = 1$, $x = -3$

$f'' = 0$: $x = -1$

inc/dec :

		-3		1	
f':	+	0	−	0	+

concavity:

		-1	
f'':	−	0	+

$f'(-4) = (-4+3)(-4-1) = - \cdot - = +$

$f'(0) = -3$

$f'(2) = (2+3)(2-1) = + \cdot +$

$f''(-2) = 2(-2+1) = -$

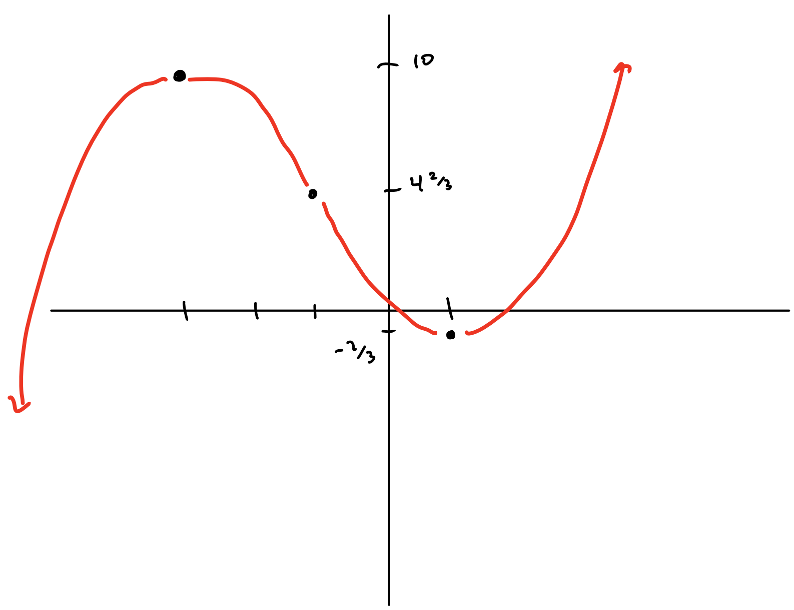
$f''(0) = 2$

y-values:

$x = -3$, $y = f(-3) = \frac{1}{3}(-3)^3 + (-3)^2 - 3(-3) + 1 = -9 + 9 + 9 + 1 = 10$ local max

$x = 1$, $y = f(1) = \frac{1}{3} \cdot 1^3 + 1^2 - 3 \cdot 1 + 1 = \frac{1}{3} + 1 - 3 + 1 = -2/3$ local min

$x = -1$, $y = f(-1) = \frac{1}{3}(-1)^3 + (-1)^2 - 3(-1) + 1 = \frac{-1}{3} + 1 + 3 + 1 = 4 + 2/3$ infl. pt

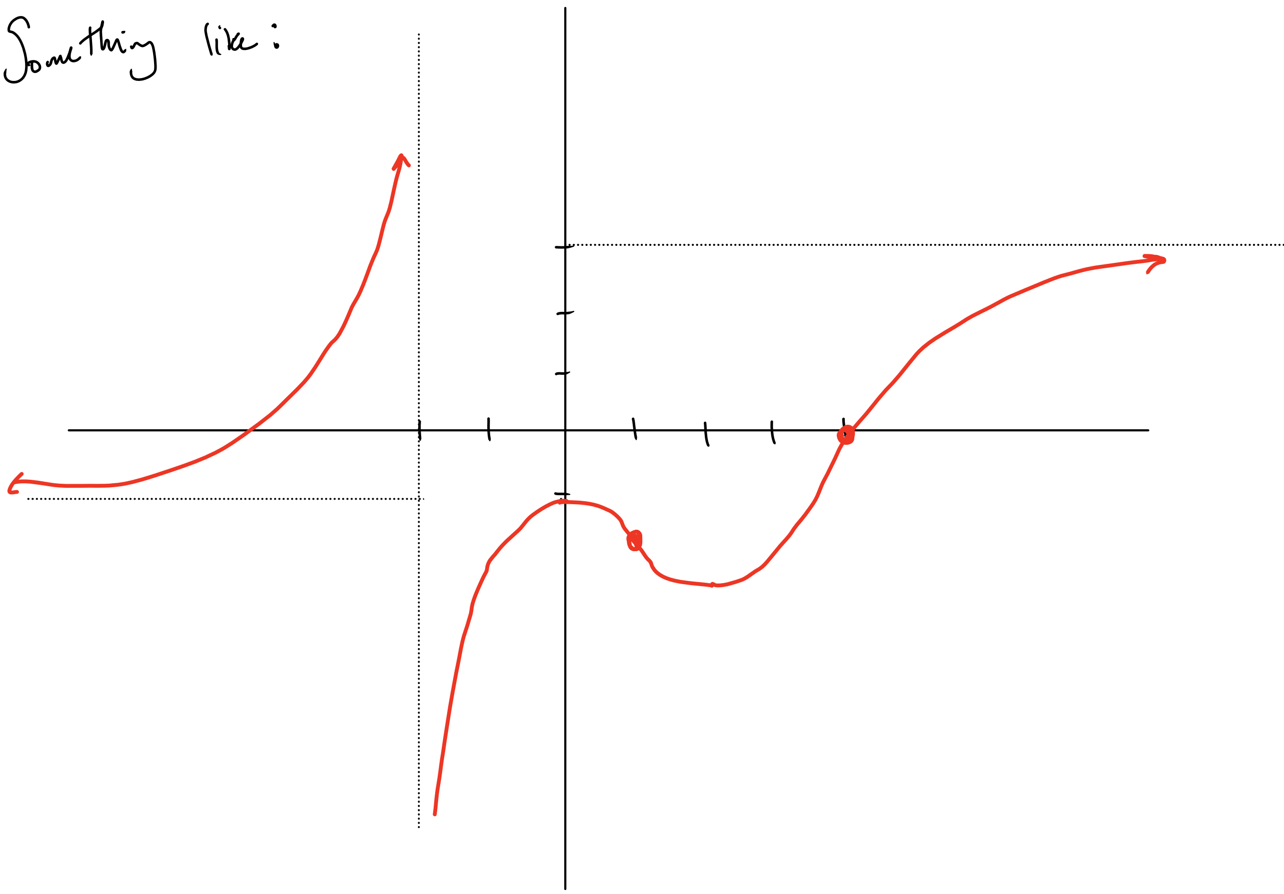


Question 8. (10 points) Please draw the graph of a function which satisfies these properties:

- Horizontal asymptotes on the right at 3, on the left at -1
- Vertical asymptote at $x = -2$
- Increasing on $(-\infty, -2)$ and $(-2, 0)$ and $(2, \infty)$
- Decreasing on $(0, 2)$
- Inflection points at $x = 1$ and $x = 4$

Put a big dot on the inflection points so I can see them.

Something like:



$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$