Math 1171

Homework #3

1.8 # 43, 53

2.1 # 5, 17

1.8 #43

$$f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \le x < 1 \\ 1/x & \text{if } x \ge 1 \end{cases}$$

$\lim_{x \to -1^-} f(x) = (-1)^2 = 1$

discontinuous at $x = -1$

$\lim_{x \to -1^+} f(x) = -1$

$\lim_{x \to 1^-} f(x) = 1$

continuous for all real #s except -1

$\lim_{x \to 1^+} f(x) = \frac{1}{1} = 1$

1.8 # 53

$f(x) = x^2 + 10 \sin x$

$f(0) = 0^2 + 10 \cdot \sin 0 = 0$

$f(100) = 100^2 + 10 \cdot \sin 100 = 10009.8\ldots$

Since $f(0) < 1000$ and $f(100) > 1000$,

there must be some c in $(0, 100)$

with $f(c) = 1000$.

2.1 #5 $\qquad y = 2x^2 - 5x + 1$ find eqn of tangent line at $(3,4)$

slope at $x = 3$:

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{2(3+h)^2 - 5(3+h) + 1 - (2\cdot 3^2 - 5\cdot 3 + 1)}{h}$$

$$= \lim_{h\to 0} \frac{2(9+6h+h^2) - 15 - 5h + 1 - (18 - 15 + 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{18} + 12h + 2h^2 - \cancel{15} - 5h + \cancel{1} - \cancel{4}}{h}$$

$$= \lim_{h\to 0} \frac{7h + 2h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(7+2h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 7 + 2h = 7$$

So the slope is 7, point $(3,4)$

$y - y_0 = m(x - x_0)$

$y - 4 = 7(x-3)$

$y - 4 = 7x - 21$

$y = 7x - 17$

2.1 #19 $\qquad f(x) = \sqrt{4x+1} \qquad a = 6$

$$f'(6) = \lim_{h\to 0} \frac{f(6+h) - f(6)}{h} = \lim_{h\to 0} \frac{\sqrt{4(6+h)+1} - \sqrt{4\cdot 6 + 1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{24+4h+1} - \sqrt{25}}{h} = \lim_{h\to 0} \frac{\sqrt{25+4h} - 5}{h} \cdot \frac{\sqrt{25+4h} + 5}{\sqrt{25+4h} + 5}$$

$$= \lim_{h\to 0} \frac{\cancel{25} + 4h - \cancel{25}}{h(\sqrt{25+4h} + 5)} = \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(\sqrt{25+4h} + 5)}$$

$$= \lim_{h\to 0} \frac{4}{\sqrt{25+4h} + 5} = \frac{4}{\sqrt{25+4\cdot 0} + 5} = \frac{4}{5+5} = \frac{4}{10} = \frac{2}{5}$$