

Math 1171

Homework #9

9, 12, 19b, 43

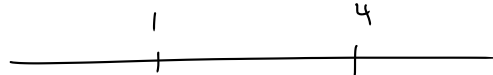
#9 $2x^3 - 15x^2 + 24x - 5$

$$f'(x) = 6x^2 - 30x + 24$$

$$= 6(x^2 - 5x + 4)$$

$$= 6(x-4)(x-1)$$

$f'_{=0}$: $x=4, x=1$



f' : $+ \quad 0 \quad - \quad 0 \quad +$

$$f'(0) = 6(0-4)(0-1) = + \cdot - \cdot - = +$$

$$f'(2) = 6(2-4)(2-1) = + \cdot - \cdot + = -$$

$$f'(5) = 6(5-4)(5-1) = + \cdot + \cdot +$$

f is increasing on $(-\infty, 1)$ & $(4, \infty)$
decreasing on $(1, 4)$

#12 $f(x) = x^{2/3}(x-3) = x^{5/3} - 3x^{2/3}$

$$f'(x) = \frac{5}{3}x^{2/3} - 3 \cdot \frac{2}{3}x^{-1/3}$$

$$= \frac{5}{3}x^{2/3} - \frac{2}{x^{1/3}}$$

$f'=0$: $\frac{5}{3}x^{2/3} - \frac{2}{x^{1/3}} = 0$ (mult by $x^{1/3}$)

$$\frac{5}{3}x - 2 = 0$$

$$\frac{5}{3}x = 2$$

$$x = 6/5$$

f'_{DNE} : $x=0$

$$f': \quad \begin{array}{c} 0 \qquad \qquad \qquad 6/5 \\ \hline + \quad \text{DNE} \quad - \quad 0 \quad + \end{array}$$

$$f'(-1) = \frac{5}{3}(-1)^{2/3} - \frac{2}{(-1)^{1/3}} = \frac{5}{3} \cdot 1 - \frac{2}{-1} = \frac{5}{3} + 2 = +$$

$$f'(1) = \dots = \frac{5}{3} \cdot 1 - \frac{2}{1} = \frac{5}{3} - 2 = -$$

$$f'(8) = \frac{5}{3}(8)^{2/3} - \frac{2}{8^{1/3}} = \frac{5}{3} \cdot 4 - \frac{2}{2} = +$$

f is increasing on $(-\infty, 0)$ & $(6/5, \infty)$
decreasing on $(0, 6/5)$

196 $f(x) = x^4 - 2x^2 + 3$

$$\begin{aligned} f'(x) &= 4x^3 - 4x \\ &= 4x(x^2 - 1) \\ &= 4x(x-1)(x+1) \end{aligned}$$

$$f' = 0 \quad x = 0, 1, -1$$

$$f''(x) = 12x^2 - 4$$

$$f''(0) = -4 \quad \text{so } \underline{x=0 \text{ is a local max}}$$

$$f''(1) = 12 - 4 = 8 \quad \text{so } \underline{x=1 \text{ is a local min}}$$

$$f''(-1) = 12(-1)^2 - 4 = 8 \quad \text{so } \underline{x=-1 \text{ is a local min}}$$

#43c

$$g(t) = 3t^4 - 8t^3 + 12$$

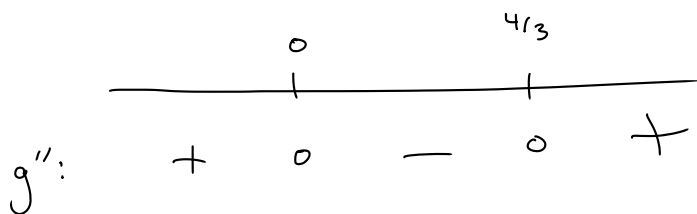
$$g'(t) = 12t^3 - 24t^2$$

$$g''(t) = 36t^2 - 48t$$

$$g'' = 0 \quad 36t^2 - 48t = 0$$

$$12t(3t - 4) = 0$$

$$t = 0 \quad t = 4/3$$



$$g''(-1) = 12(-1)(3(-1) - 4) = +$$

$$g''(1) = 12 \cdot 1 \cdot (3 - 4) = -$$

$$g''(2) = 12 \cdot 2 \cdot (3 \cdot 2 - 4) = +$$

So g is concave up on $(-\infty, 0)$ & $(4/3, \infty)$
concave down on $(0, 4/3)$