Math 1171

Homework #9

9, 12, 19b, 43

#9 $2x^3 - 15x^2 + 24x - 5$

$f'(x) = 6x^2 - 30x + 24$

$= 6(x^2 - 5x + 4)$

$= 6(x-4)(x-1)$

$\underline{f'=0}$: $x=4$, $x=1$

		1		4	
f':	$+$	0	$-$	0	$+$

$f'(0) = 6(0-4)(0-1) = + \cdot - \cdot - = +$

$f'(2) = 6(2-4)(2-1) = + \cdot - \cdot + = -$

$f'(5) = 6(5-4)(5-1) = + \cdot + \cdot +$

f is increasing on $(-\infty, 1)$ & $(4, \infty)$

decreasing on $(1, 4)$

#12 $f(x) = x^{2/3}(x-3) = x^{5/3} - 3x^{2/3}$

$f'(x) = \frac{5}{3}x^{2/3} - 3 \cdot \frac{2}{3}x^{-1/3}$

$= \frac{5}{3}x^{2/3} - \frac{2}{x^{1/3}}$

$\underline{f'=0}$: $\frac{5}{3}x^{2/3} - \frac{2}{x^{1/3}} = 0$ (mult by $x^{1/3}$)

$\frac{5}{3}x - 2 = 0$

$5/3\, x = 2$

$x = 6/5$

$\underline{f'\text{DNE}}$: $x = 0$

$$\begin{array}{ccccccc} & & 0 & & \tfrac{6}{5} & & \\ f': & + & \text{DNE} & - & 0 & + \end{array}$$

$f'(-1) = \frac{5}{3}(-1)^{2/3} - \frac{2}{(-1)^{1/3}} = \frac{5}{3} \cdot 1 - \frac{2}{-1} = 5/3 + 2 = +$

$f'(1) = \ldots\ldots = \frac{5}{3} \cdot 1 - \frac{2}{1} = 5/3 - 2 = -$

$f'(8) = \frac{5}{3}(8)^{2/3} - \frac{2}{8^{1/3}} = \frac{5}{3} \cdot 4 - \frac{2}{2} = +$

f is increasing on $(-\infty, 0)$ & $(6/5, \infty)$

decreasing on $(0, 6/5)$

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$f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x-1)(x+1)$

$\underline{f' = 0}$ $\quad x = 0, 1, -1$

$f''(x) = 12x^2 - 4$

$f''(0) = -4$ so $\underline{x = 0 \text{ is a local max}}$

$f''(1) = 12 - 4 = 8$ so $\underline{x = 1 \text{ is a local min}}$

$f''(-1) = 12(-1)^2 - 4 = 8$ so $\underline{x = -1 \text{ is a local min}}$

#43c

$g(t) = 3t^4 - 8t^3 + 12$

$g'(t) = 12t^3 - 24t^2$

$g''(t) = 36t^2 - 48t$

$\underline{g''=0}$ $\quad 36t^2 - 48t = 0$

$12t(3t-4) = 0$

$t = 0 \qquad t = 4/3$

		0		$4/3$	
g'':	$+$	0	$-$	0	$+$

$g''(-1) = \underbrace{12}_{+}\underbrace{(-1)}_{-}\underbrace{(3(-1)-4)}_{-} = +$

$g''(1) = \underbrace{12}_{+}\cdot\underbrace{1}_{+}\cdot\underbrace{(3-4)}_{-} = -$

$g''(2) = \underbrace{12}_{+}\cdot\underbrace{2}_{+}\cdot\underbrace{(3\cdot 2-4)}_{+} = +$

So g is concave up on $(-\infty, 0)$ & $(4/3, \infty)$

concave down on $(0, 4/3)$