Math 1171

Homework #10

3.4 # 11, 40

3.5 # 9

3.7 # 40

3.4 #11

$$\lim_{t\to-\infty} \frac{3t^2+t}{t^3-4t+1} = \lim_{t\to-\infty} \frac{3\frac{t^2}{t^3}+\frac{t}{t^3}}{\frac{t^3}{t^3}-4\frac{t}{t^3}+\frac{1}{t^3}}$$

$$= \lim_{t\to-\infty} \frac{3/t + 1/t^2}{1-4\cdot\frac{1}{t^2}+\frac{1}{t^3}} = \frac{0+0}{1-0+0} = 0$$

3.4 #40

$$y = \frac{x-9}{\sqrt{4x^2+3x+2}}$$

vert 'totes

$$\sqrt{4x^2+3x+2} = 0$$

$$4x^2+3x+2 = 0$$

quad. form. gives no solutions!

[$b^2-4ac = 9-4\cdot4\cdot12$ is negative] So **no vert 'totes**

horz 'totes

$$\lim_{x\to\infty} \frac{x-9}{\sqrt{4x^2+3x+2}} = \lim_{x\to\infty} \frac{\frac{x}{x}-\frac{9}{x}}{\sqrt{\frac{4x^2}{x^2}+\frac{3x}{x^2}+\frac{2}{x^2}}} = \lim_{x\to\infty} \frac{1-\frac{9}{x}}{\sqrt{4+\frac{3}{x}+\frac{2}{x^2}}}$$

$$= \frac{1-0}{\sqrt{4+0+0}} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{x-9}{\sqrt{4x^2+3x+2}} = \lim_{x\to-\infty} \frac{\frac{x}{x}-\frac{9}{x}}{-\sqrt{\frac{4x^2}{x^2}+\frac{3x}{x^2}+\frac{2}{x^2}}}$$

$$= \dots = -\frac{1}{2}$$

3.5 #9 Graph $y = \frac{2x+3}{x+2}$

vert tote: $x+2=0$
$x=-2$

horz: $\lim_{x\to\infty} \frac{2x+3}{x+2} = \lim_{x\to\infty} \frac{\frac{2x}{x}+\frac{3}{x}}{\frac{x}{x}+\frac{2}{x}} = \lim_{x\to\infty} \frac{2+\frac{3}{x}}{1+\frac{2}{x}} = 2$

$\lim_{x\to -\infty}$ is also 2.

critical #s
$f'(x) = \frac{(x+2)(2)-(2x+3)\cdot 1}{(x+2)^2} = \frac{2x+4-2x-3}{(x+2)^2} = \frac{1}{(x+2)^2}$

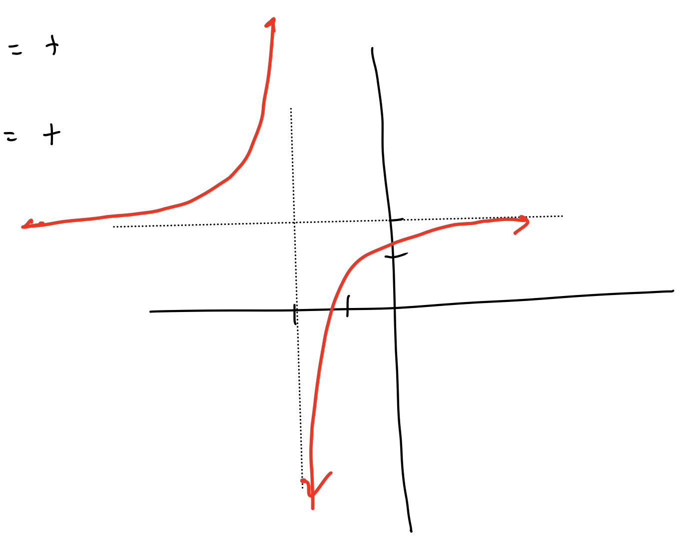
$f'=0$: $\frac{1}{(x+2)^2}=0$
$\cancel{1=0}$
so f' is never 0.

f' DNE $(x+2)^2=0$
$x=-2$

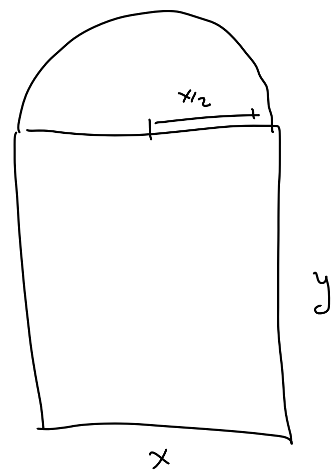
inc/dec

$$\begin{array}{c|ccccc} & & & -2 & & \\ \hline f' & & + & \text{DNE} & + & \end{array}$$

$f'(-3) = \frac{1}{(\ )^2} = +$
$f'(0) = \frac{1}{(\ )^2} = +$



3.7 #40



Maximize the area:

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$A = xy + \frac{1}{8}\pi x^2$$

The perimeter is 30:

top part

$$30 = x + 2y + \frac{1}{2}\pi x$$

so $2y = 30 - x - \frac{\pi}{2}x$

$$y = 15 - \frac{x}{2} - \frac{\pi}{4}x$$

So $A(x) = x\left(15 - \frac{x}{2} - \frac{\pi}{4}x\right) + \frac{1}{8}\pi x^2$

$$= 15x - \frac{x^2}{2} - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2$$

$$A(x) = 15x - \frac{x^2}{2} - \frac{\pi}{8}x^2$$

interval: smallest x: 0

biggest x: 30 (actually $x = 30$ is impossible, but 30 is still an upper bound)

Critical #s: $A'(x) = 15 - x - \frac{\pi}{4}x$

$A' = 0$: $15 - x - \frac{\pi}{4}x = 0$

$$15 = x + \frac{\pi}{4}x$$

$$x = \frac{15}{1 + \pi/4} = 8.4\ldots$$

$$= \frac{60}{4 + \pi}$$

Plug in:

$$A(0) = 0$$

$$A(30) = 15 \cdot 30 - \frac{30^2}{2} - \frac{\pi}{8} \cdot 30^2 = \text{negative}$$

$$A(8.4) = 15 \cdot 8.4 - \frac{8.4^2}{2} - \frac{\pi}{8} \cdot 8.4^2 = 63.0$$

To get maximum area, use $x \approx 8.4$

and $y = 15 - \frac{x}{2} - \frac{\pi}{4}x \approx 4.2$