

Math 1171

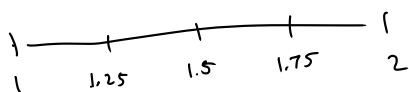
Homework #12

Section 4.1 #3

Section 4.2 #31

Section 4.3 #19, 35

4.1 #3 $f(x) = \frac{1}{x}$ on $[1, 2]$, $\Delta x = .25$



$$R_4 = \frac{1}{1.25} \cdot .25 + \frac{1}{1.5} \cdot .25 + \frac{1}{1.75} \cdot .25 + \frac{1}{2} \cdot .25$$

$$L_4 = \frac{1}{1} \cdot .25 + \frac{1}{1.25} \cdot .25 + \frac{1}{1.5} \cdot .25 + \frac{1}{1.75} \cdot .25$$

4.2 #31 $\int_1^5 3x^2 + 7x \, dx$ $\Delta x = \frac{5-1}{n} = \frac{4}{n}$

$$x_i = 1 + \frac{4i}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 \left(1 + \frac{4i}{n} \right)^2 + 7 \left(1 + \frac{4i}{n} \right) \right) \frac{4}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 \left(1 + \frac{8i}{n} + \frac{16i^2}{n^2} \right) + 7 + \frac{28i}{n} \right) \frac{4}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 + \frac{24i}{n} + \frac{48i^2}{n^2} + 7 + \frac{28i}{n} \right) \frac{4}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(10 + \frac{52i}{n} + \frac{48i^2}{n^2} \right) \frac{4}{n}$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{40}{n} + \frac{208}{n^2} i + \frac{192}{n^3} i^2$$

$$\begin{aligned}
&= \lim_{n \rightarrow \infty} \frac{40}{n} \sum_{i=1}^n 1 + \frac{208}{n^2} \sum_{i=1}^n i + \frac{192}{n^3} \sum_{i=1}^n i^2 \\
&= \lim_{n \rightarrow \infty} \frac{40}{n} \cdot n + \frac{208}{n^2} \cdot \frac{n(n+1)}{2} + \frac{192}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \\
&= 40 + 208 \cdot \frac{1}{2} + 192 \cdot \frac{1}{3} \\
&= \underline{208}
\end{aligned}$$

4.3 #18

$$y = \int_0^{x^4} \cos^2 \theta \, d\theta$$

$$y' = \cos^2(x^4) \cdot 4x^3$$

4.3 #35

$$\int_0^1 (u+2)(u-3) \, du = \int_0^1 u^2 - u - 6 \, du$$

$$= \left. \frac{1}{3}u^3 - \frac{1}{2}u^2 - 6u \right|_0^1$$

$$= \frac{1}{3} \cdot 1^3 - \frac{1}{2} \cdot 1^2 - 6 \cdot 1 - \left(\frac{1}{3} \cdot 0^3 - \frac{1}{2} \cdot 0^2 - 6 \cdot 0 \right)$$