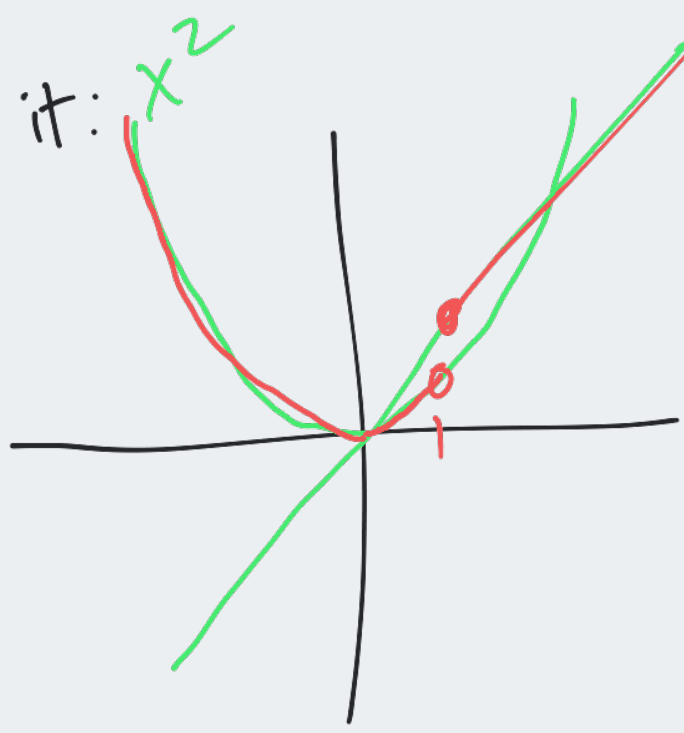
Piecewise functions

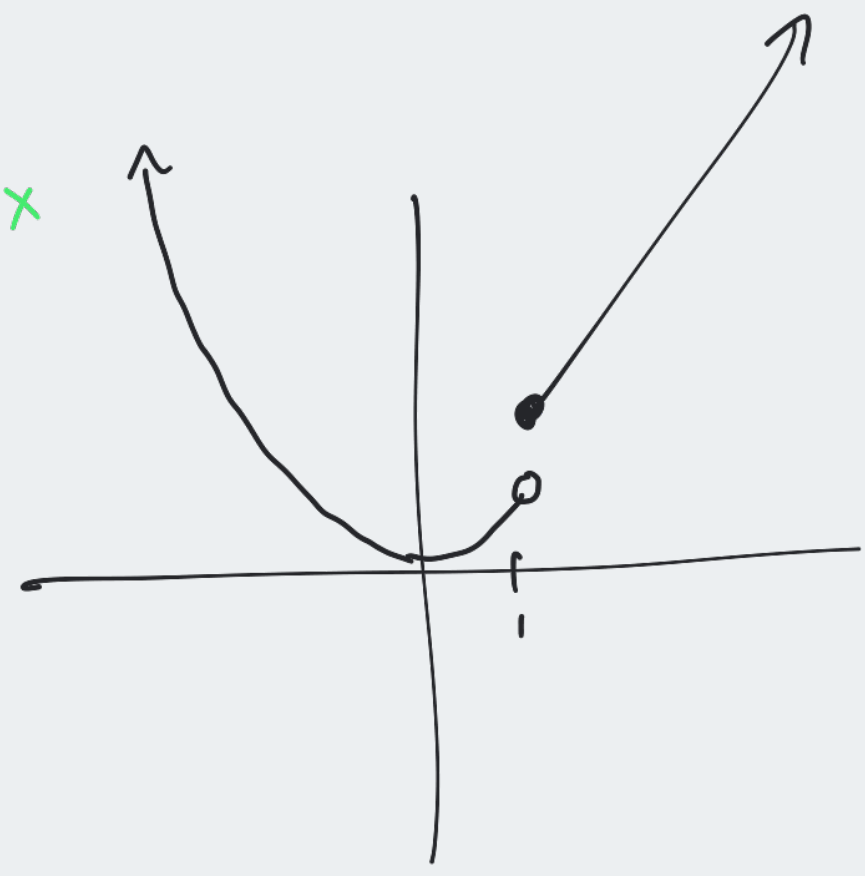
$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 5x & \text{if } x \geq 1 \end{cases}$$

$f(0) = 0^2 = 0$ $\quad$ $f(1) = 5 \cdot 1 = 5$

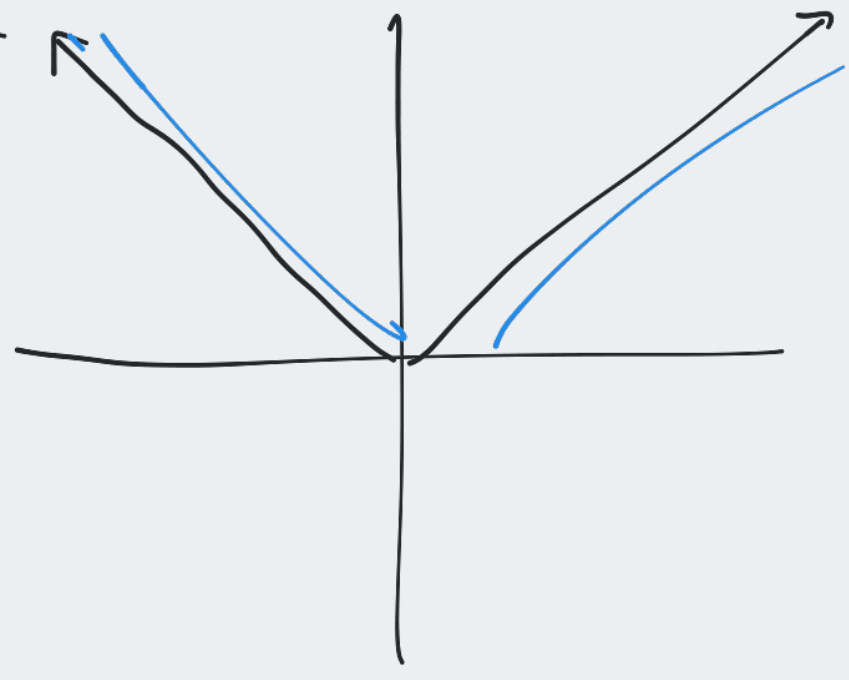
$f(4) = 5 \cdot 4 = 20$

Graph it:



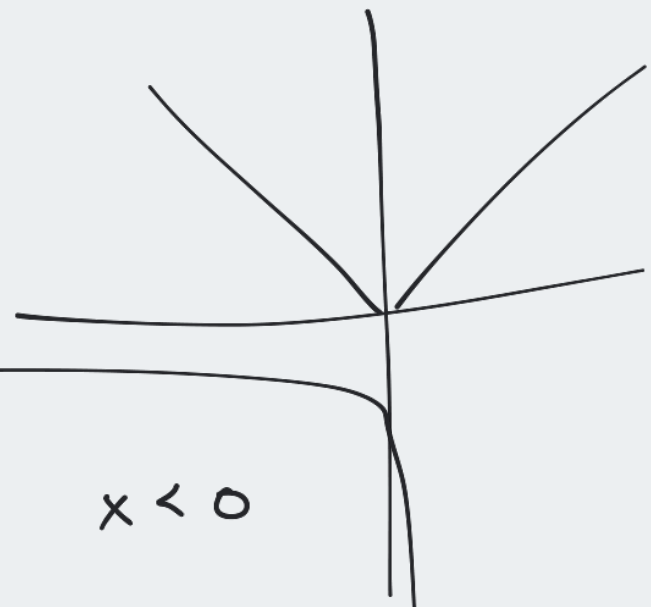


Absolute value function



we can write it in pieces

The real definition:



$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Essential functions

Linear
Polynomials
Powers
Rational
Algebraic

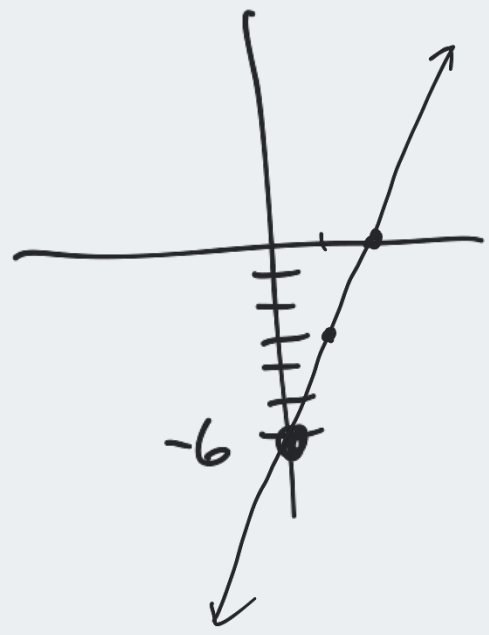
Trigonometric
Exponential
Logarithmic
} in calc 2.

Linear

$$y = mx + b$$ slope-intercept form

slope: m ↑ y-intercept: b

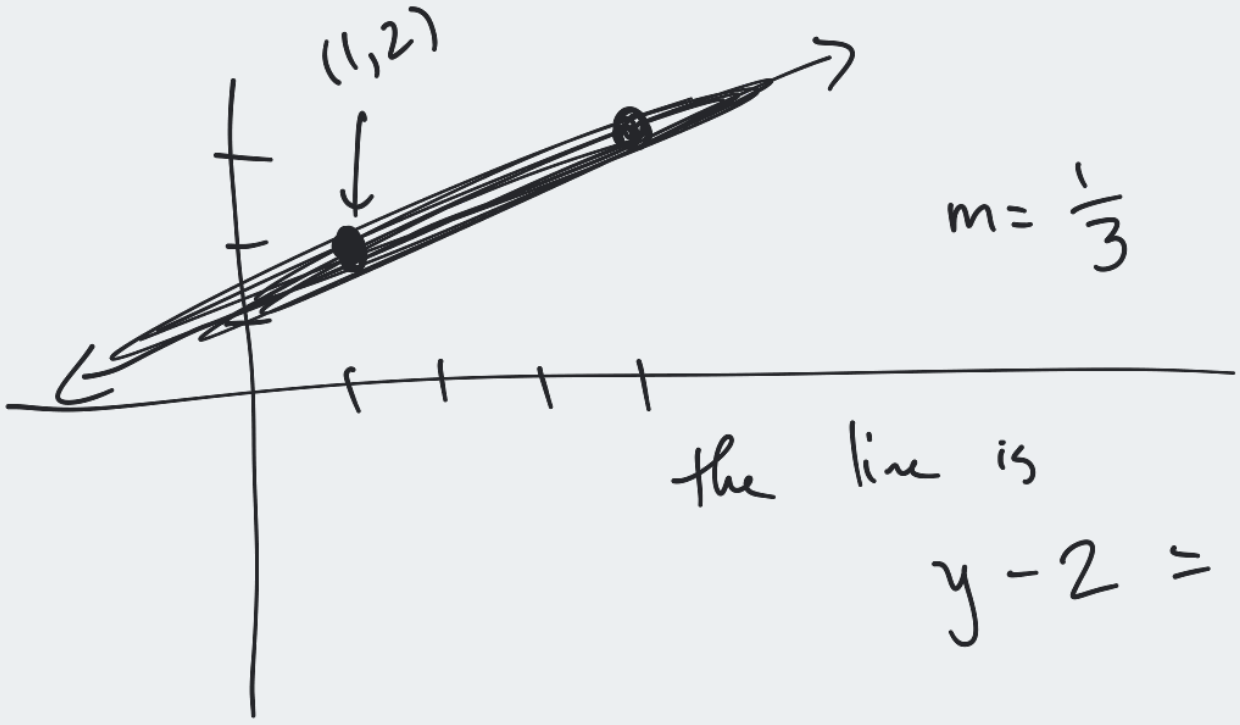
$$y = 3x - 6$$



point-slope form

$$y - y_0 = m(x - x_0)$$

m: slope and (x_0, y_0) is a point on the line.



$m = \frac{1}{3}$ $(x_0, y_0) = (1, 2)$

the line is

$$y - 2 = \frac{1}{3}(x - 1)$$

$$y - 2 = \frac{1}{3}(x - 1)$$

$$y - 2 = \frac{1}{3}x - \frac{1}{3}$$

$$y = \frac{1}{3}x - \frac{1}{3} + 2 = \frac{1}{3}x + 5/3$$

Polynomials

for $8x^5 + 7x^3 - 2x^2 + x - 7$

powers must be positive integers.

degree 5

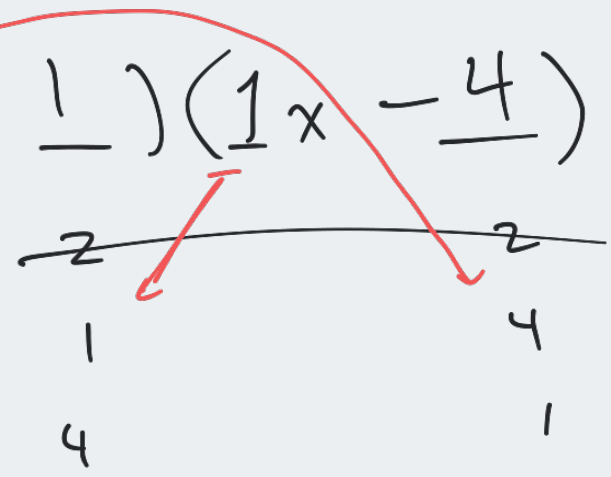
Factoring a quadratic:

$$x^2 + 7x + 12 = (x + 4)(x + 3)$$

$$x^2 - 8x + 7 = (x - 7)(x - 1)$$

$$x^2 - x - 2 = (x - 2)(x + 1)$$

$3x^2 - 11x - 4 = (3x + \underline{1})(1x - \underline{4})$



$= (3x+1)(x-4)$

---

can solve for x by factoring:

solve for x in $2x^2 + 4 = -9x$

$2x^2 + 9x + 4 = 0$

factor $(2x + 1)\cdot(x + 4) = 0$

$2x+1 = 0$ or $x+4 = 0$

$x = -1/2$ or $x = -4$

---

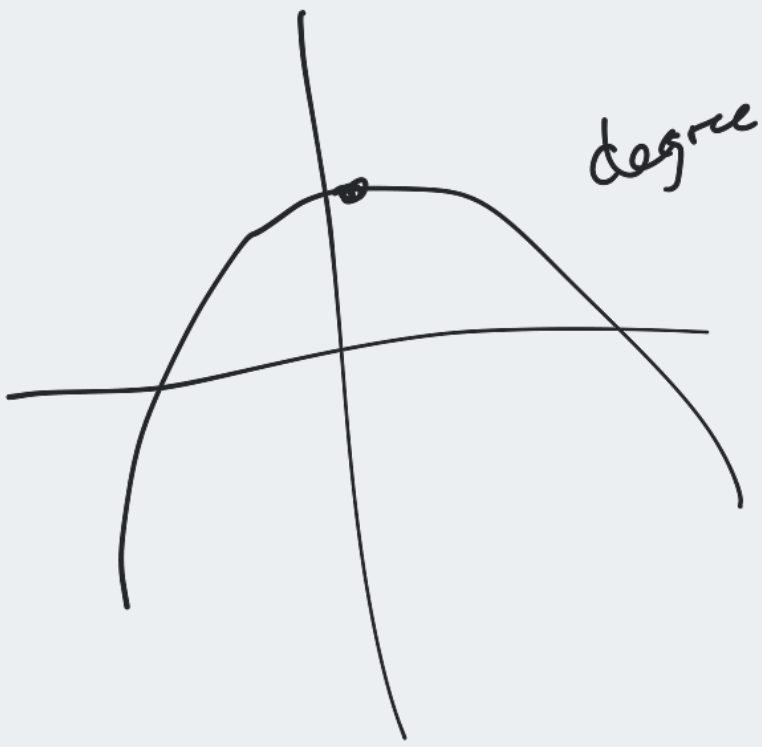
quadratic formula:

$$ax^2 + bx + c = 0$$

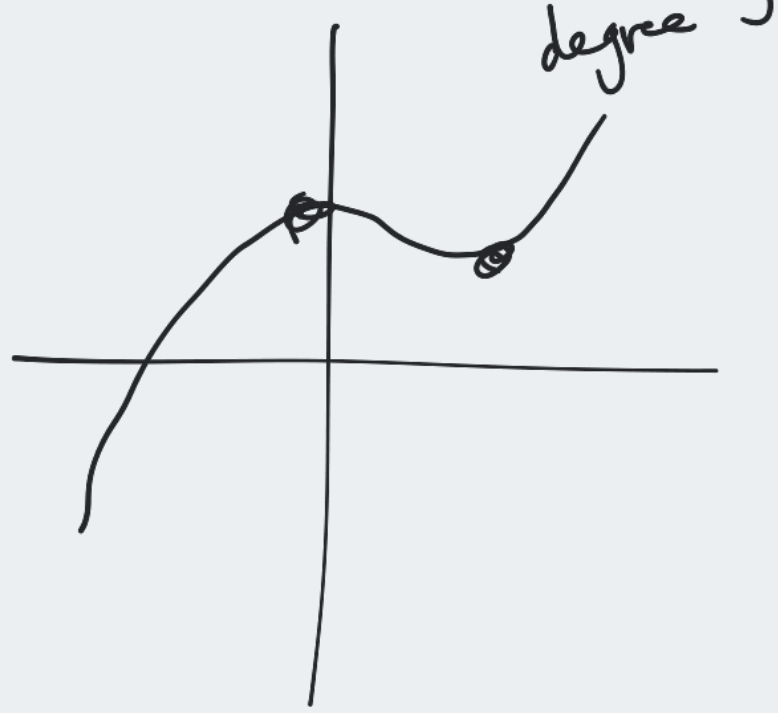
solutions are: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphs of polynomials look like curves with some # of "turns",

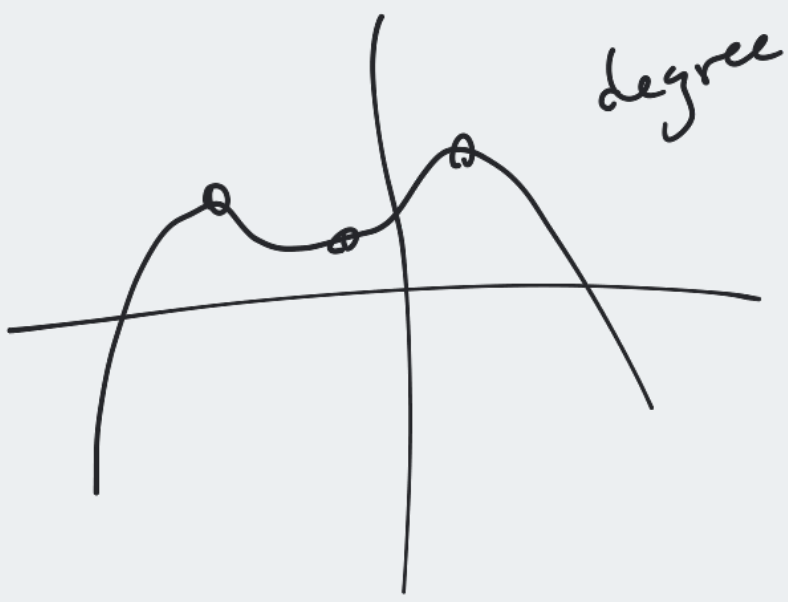
# of turns is deg − 1.



degree 2.



degree 3



degree 4.

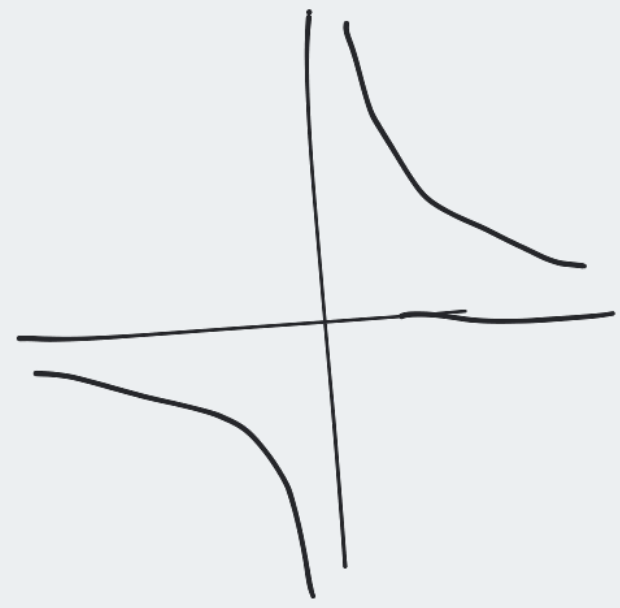
etc.

Powers

$f(x) = x^n$

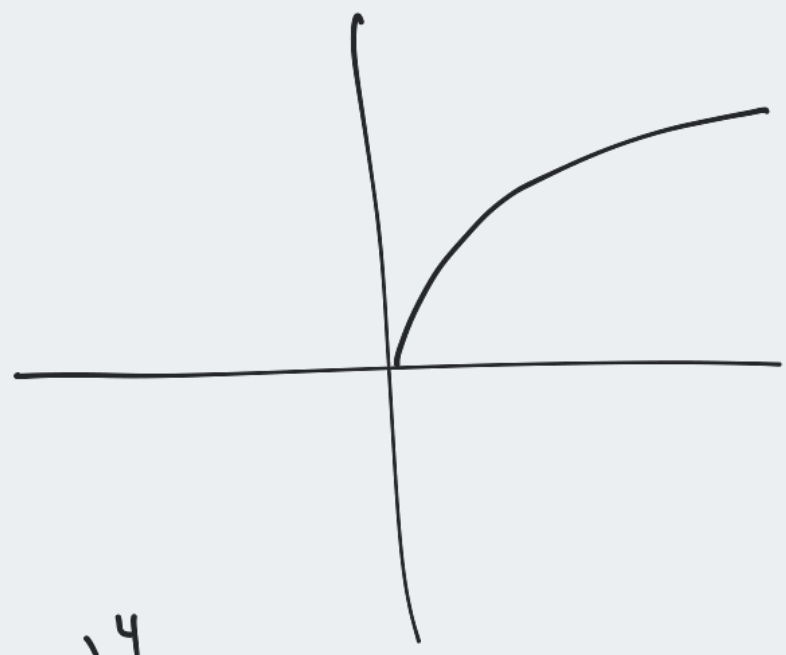
$n = -1$:

$f(x) = x^{-1} = \frac{1}{x}$



generally: $x^{-n} = \frac{1}{x^n}$

$n = 1/2$ $x^{1/2} = \sqrt{x}$



$x^{1/3} = \sqrt[3]{x}$

$x^{4/7} = \sqrt[7]{x^4} = \left(\sqrt[7]{x}\right)^4$

Rational Functions

any ratio of polynomials

$$f(x) = \frac{x^3 - 2x^2 + x - 3}{x^2 - 4}$$

looks like a curve with v. asymptotes