Algebraic functions

Any combination of polynomials & powers

$$3x^2 + \sqrt{x}$$

$$3x^2 + \sqrt{x} + \frac{8x}{5x + \sqrt[3]{x}}$$

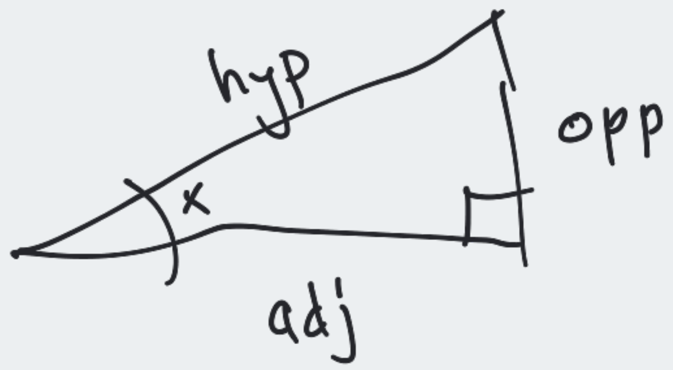
$$\sqrt{1 + \sqrt{1 + \sqrt{1+x}}}$$

not e^x or $\sin x$

Trig functions

sohcahtoa

based on

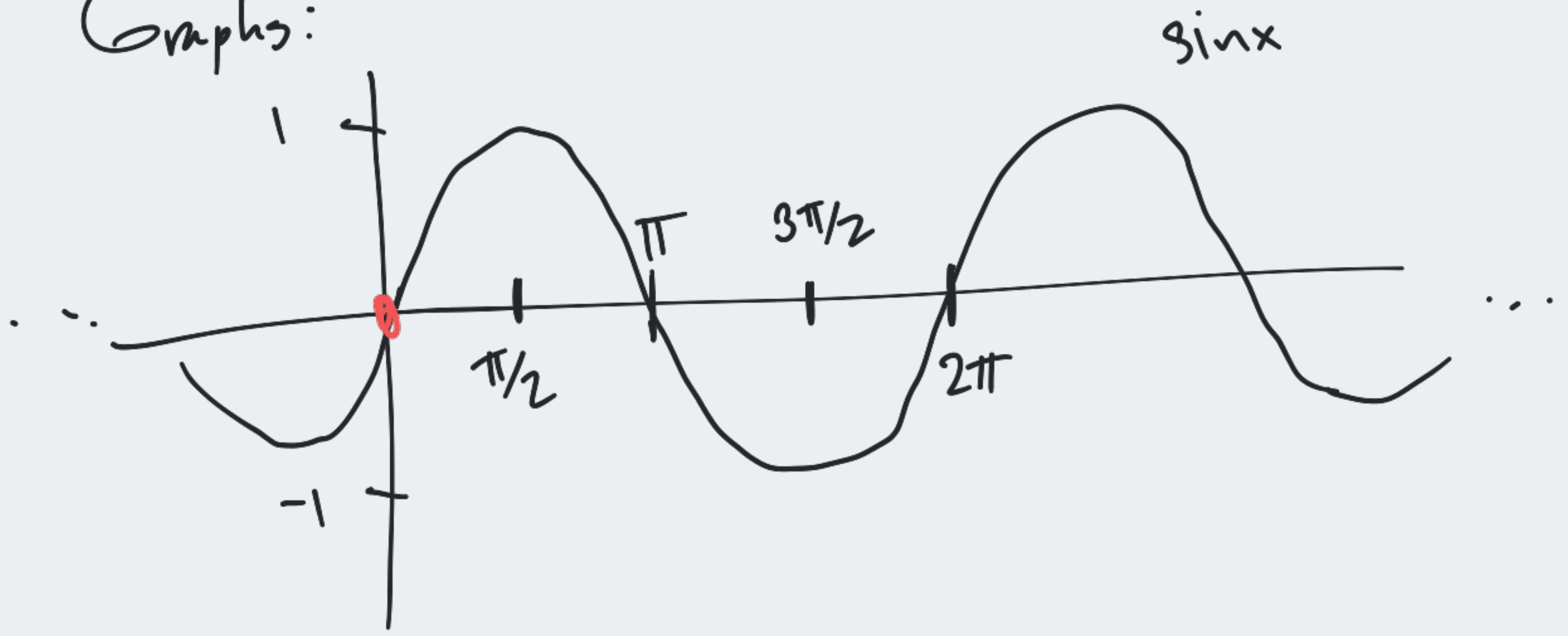


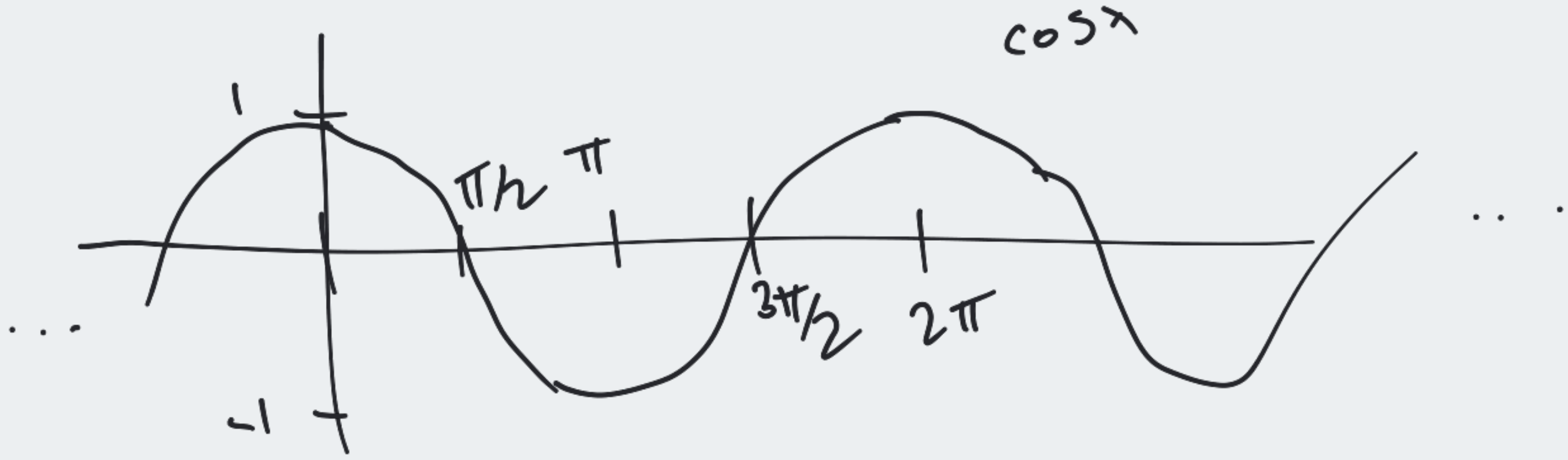
$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\tan x = \frac{\text{opp}}{\text{adj}}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

Graphs:





$\sin 0 = 0$ $\cos 0 = 1$

$\sin \pi/2 = 1$ $\cos \pi/2 = 0$

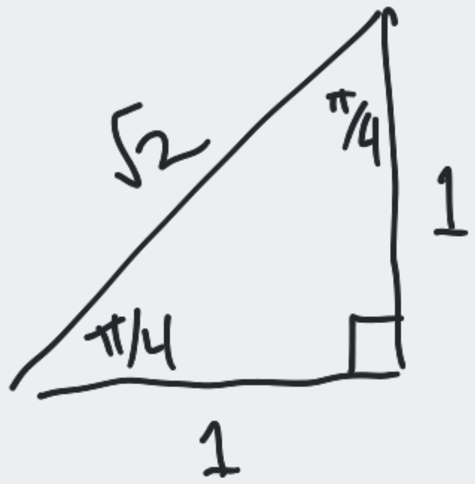
$\sin \pi = 0$ $\cos \pi = -1$

$\sin 3\pi/2 = -1$ $\cos \frac{3\pi}{2} = 0$

$\sin 2\pi = 0$ $\cos 2\pi = 1$

other values:

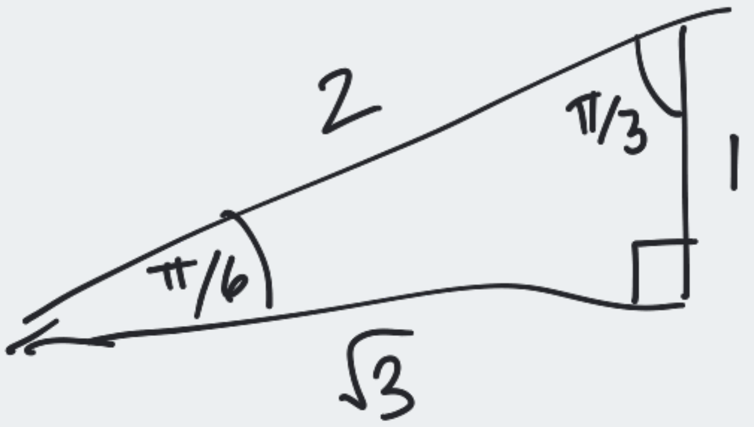
45-45-90:



$\sin \pi/4 = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

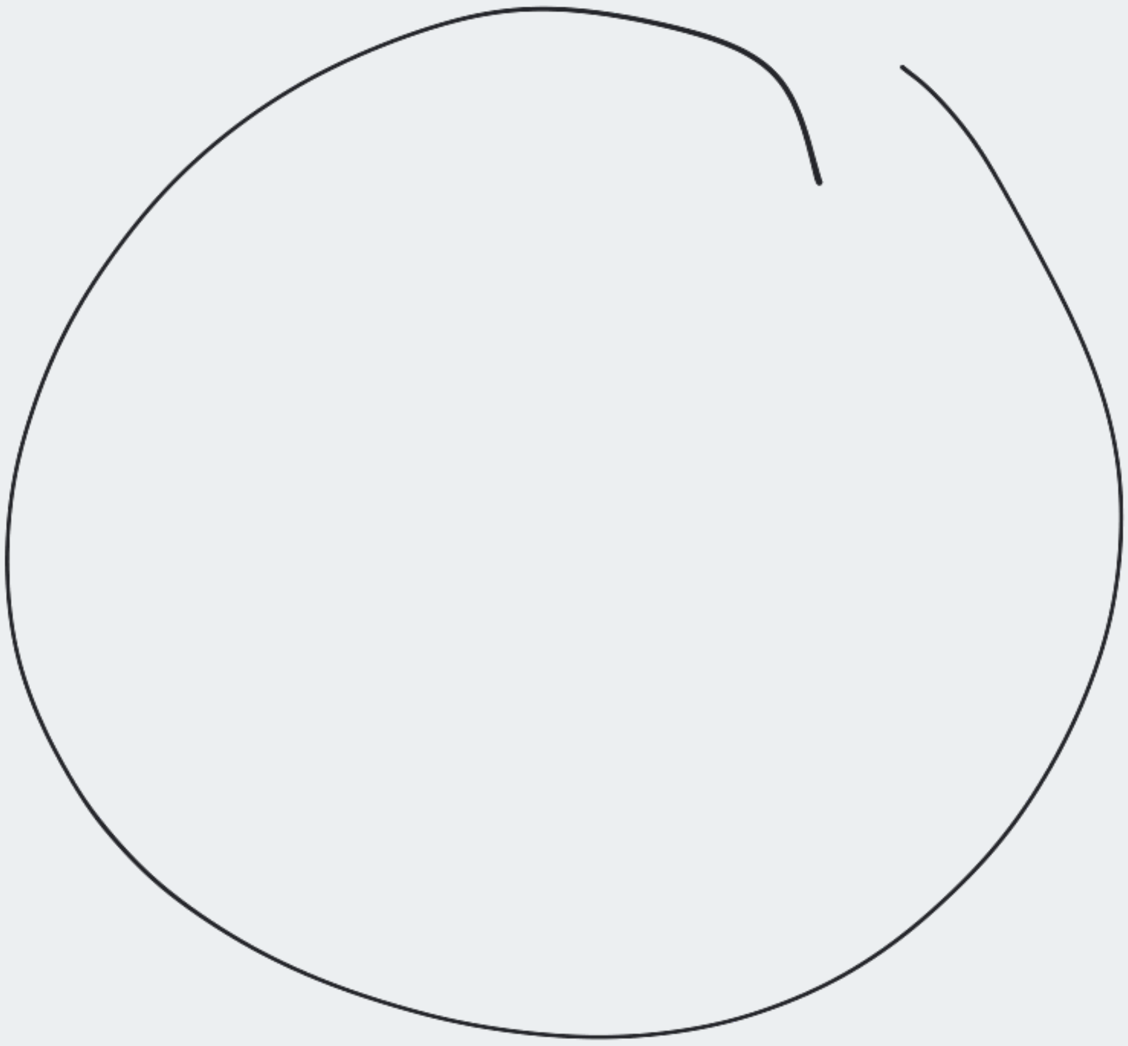
$\cos \pi/4 = \frac{1}{\sqrt{2}}$

30-60-90:

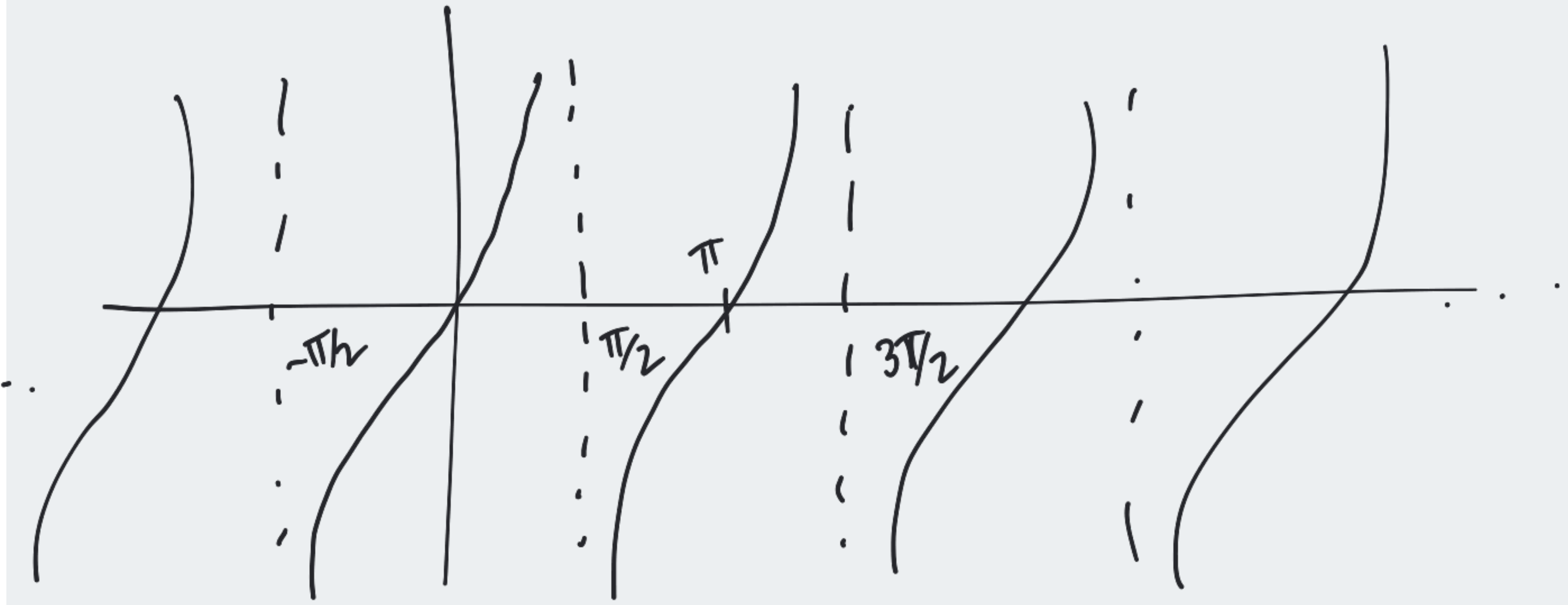


$\sin \pi/3 = \sqrt{3}/2$ $\qquad \sin \pi/6 = 1/2$

$\cos \pi/3 = 1/2$ $\qquad \cos \pi/6 = \frac{\sqrt{3}}{2}$



$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{\sin x}{\cos x}$$



Those other ones!

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

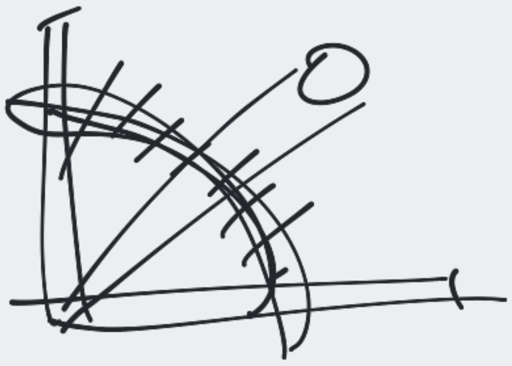
$$(\sin x)^2 + (\cos x)^2 = 1$$

$$\sin^2 x + \cos^2 x = 1$$

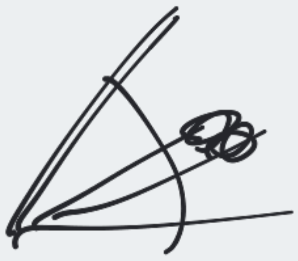
$$\sin(a+b) = \ldots$$

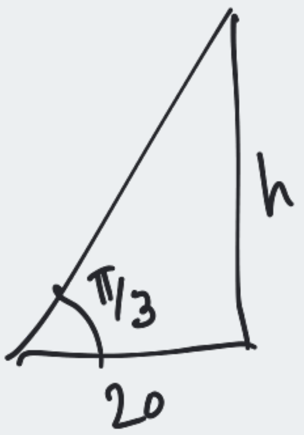
Use them to do geometry with triangles

solve for h.



sextant





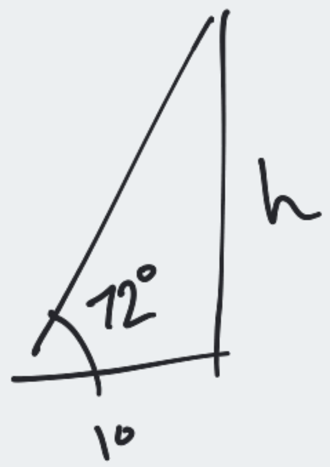
$$\tan \pi/3 = \frac{\text{opp}}{\text{adj}} = \frac{h}{20}$$

$$\frac{\sin \pi/3}{\cos \pi/3} = \frac{h}{20}$$

$$\frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = h/20$$

$$\sqrt{3} = h/20$$

$$\boxed{20\sqrt{3} = h}$$



$$\tan 72^\circ = \frac{h}{10}$$

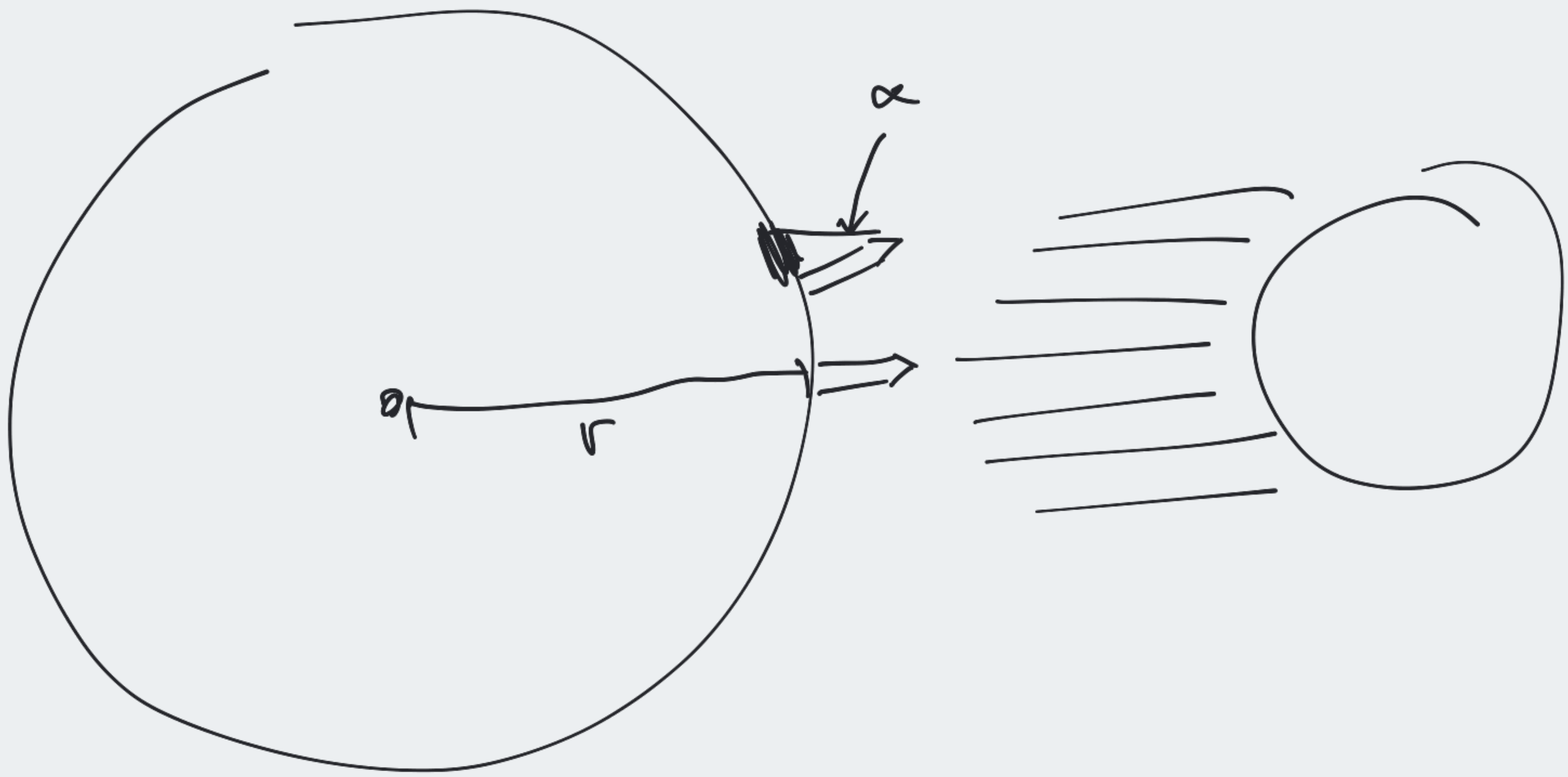
$$\downarrow$$

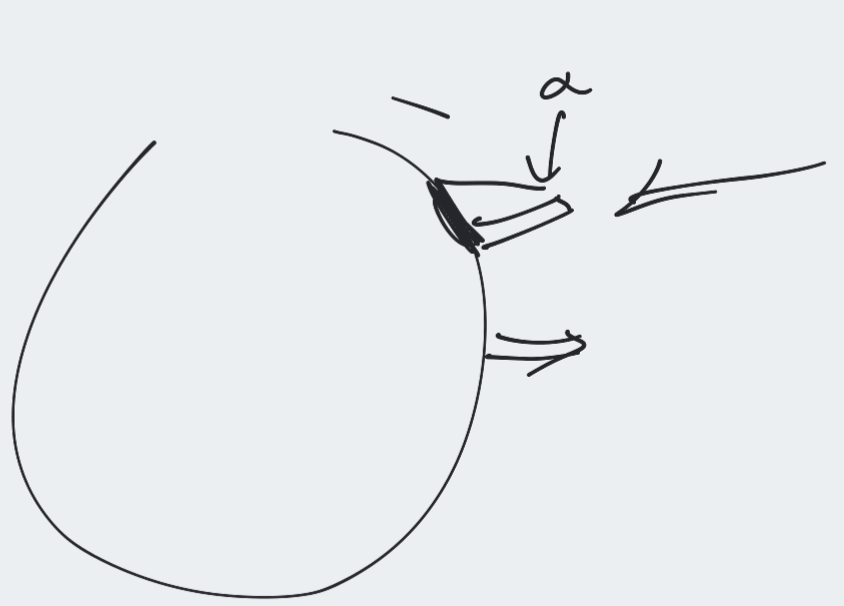
$$3.077 = h/10$$

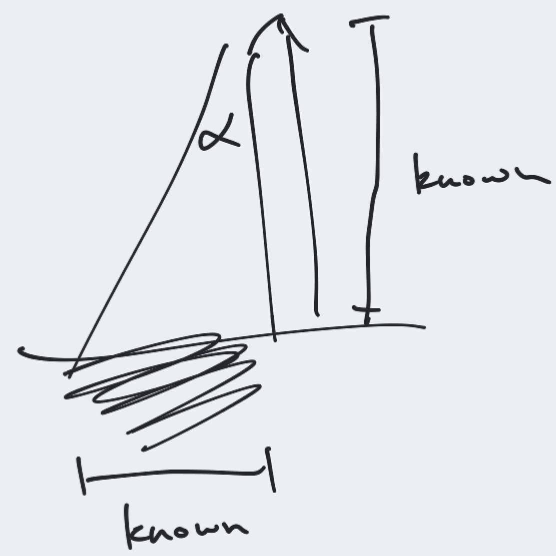
$$h = 30.77$$

First calculation of the radius of the Earth

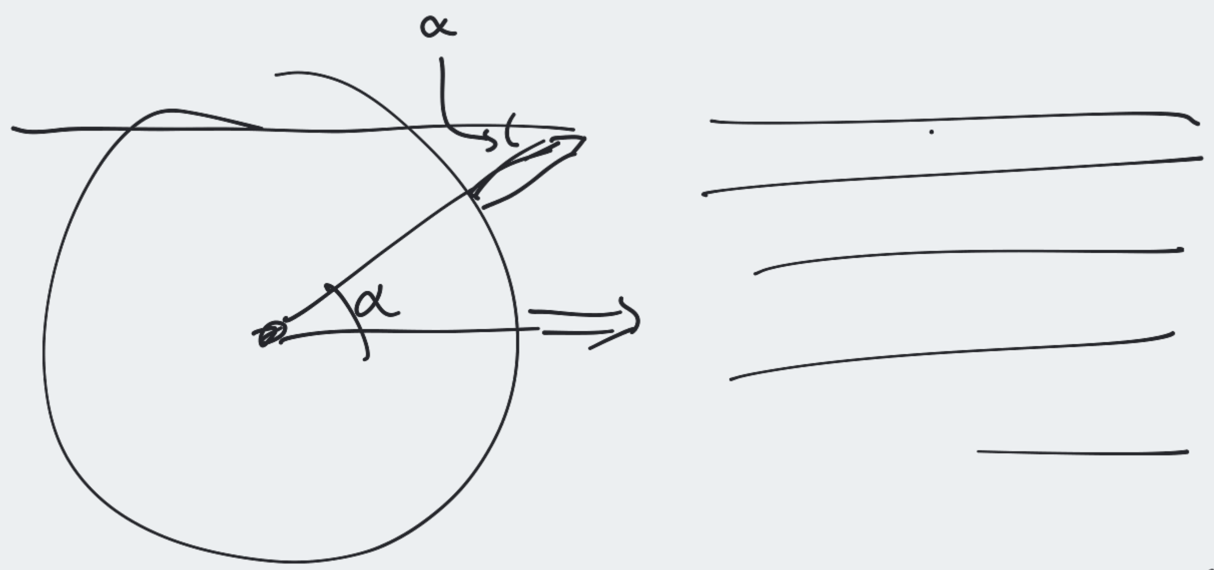
Eratosthenes ~200s BC

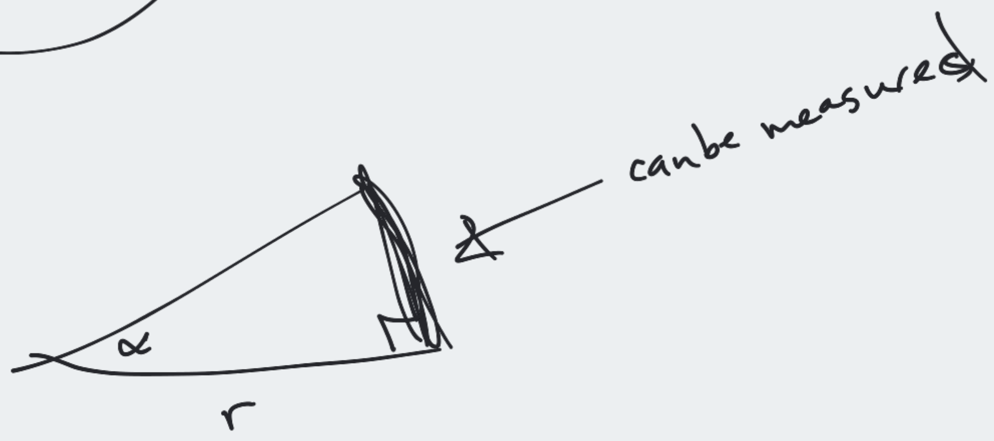






so we can find $\alpha = 7°$





He measured $r = 39,375$ km

actual: $r = 40,076$ km

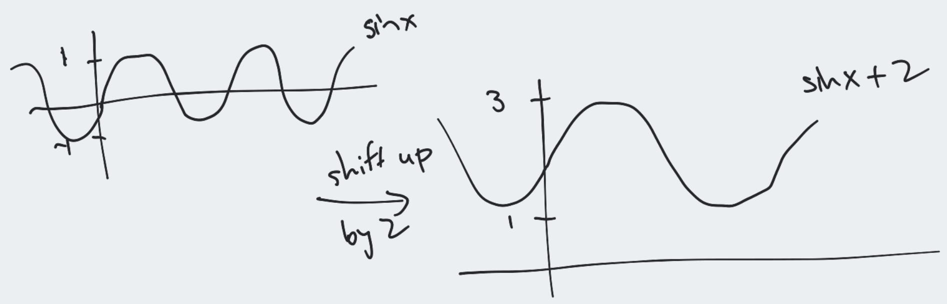
Transformations of functions

If I know the graph of $f(x)$,
we can transform it:

$f(x)+c$ is a vertical shift up by c
$f(x)-c$ . . . . down . . .

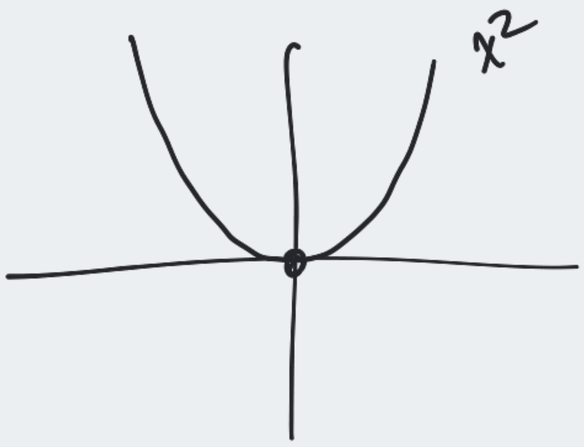
$f(x+c)$ is a horz. shift left by c
$f(x-c)$ . . . . right . . . .

graph $y = \sin x + 2$

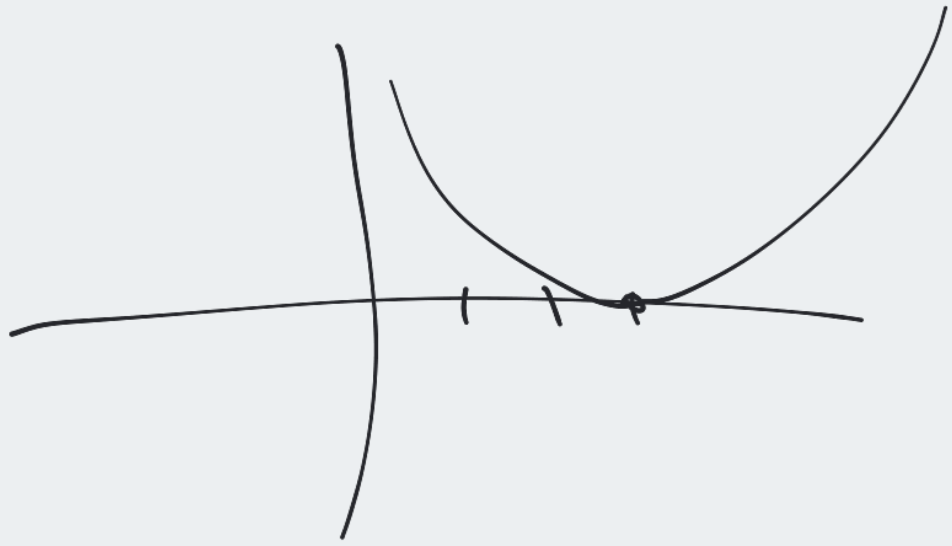


$y = (x-3)^2 \to (x-3)(x-3) = \boxed{x^2 - 6x + 9}$

this is x^2, shifted right by 3



$\xrightarrow[\text{right 3}]{\text{shift}}$



More:

$y = c \cdot f(x)$ stretches vertically by a factor of c

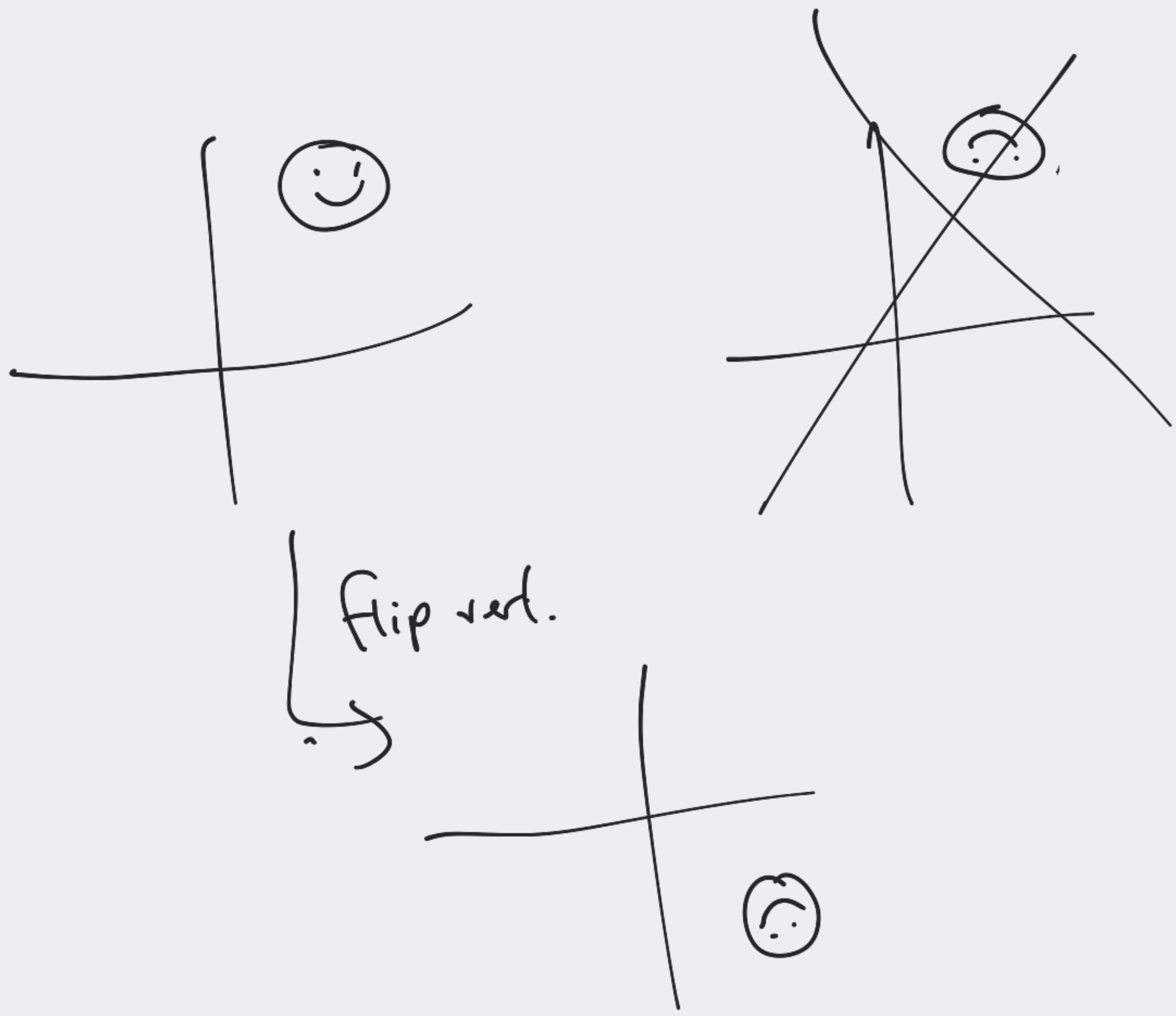
$y = f(x)/c$ compression - - - - - - -

if $c<0$, this also flips the picture vertically

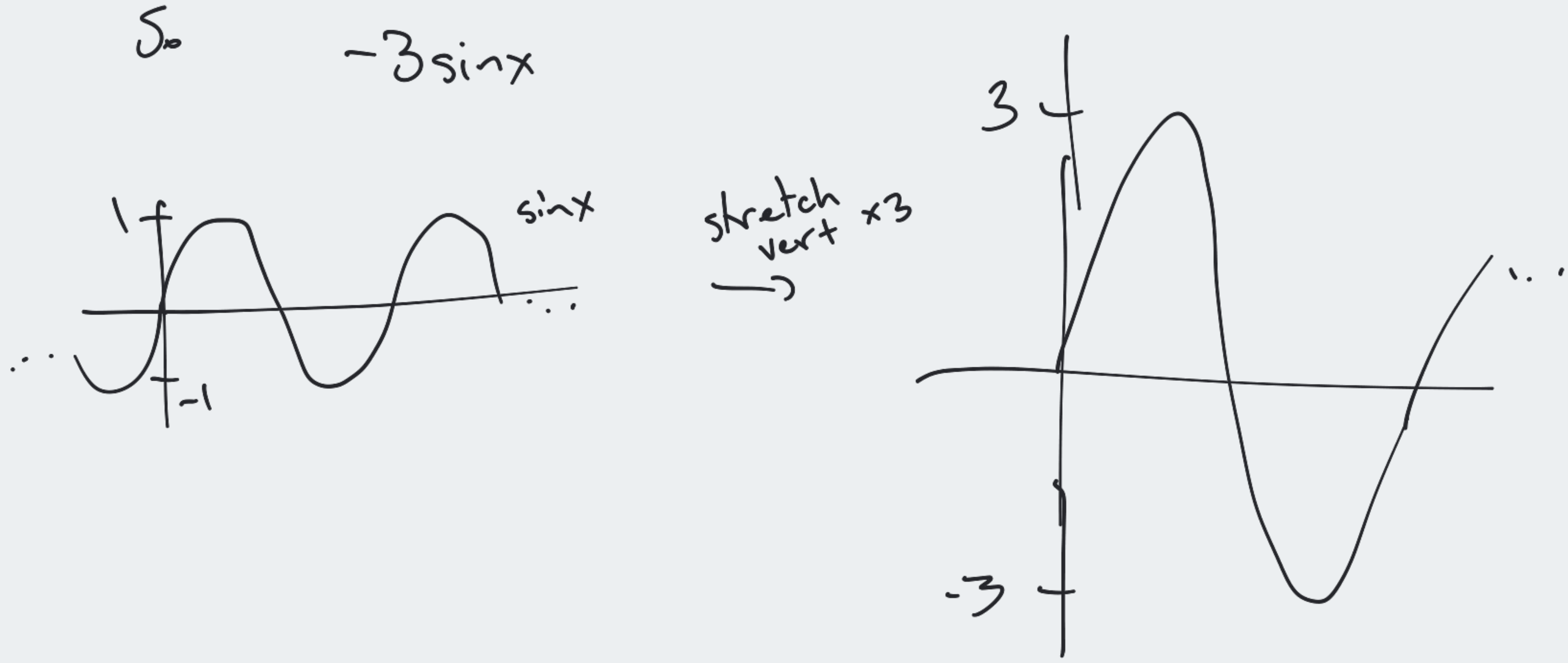
$y = f(cx)$ horz compression by a factor of c

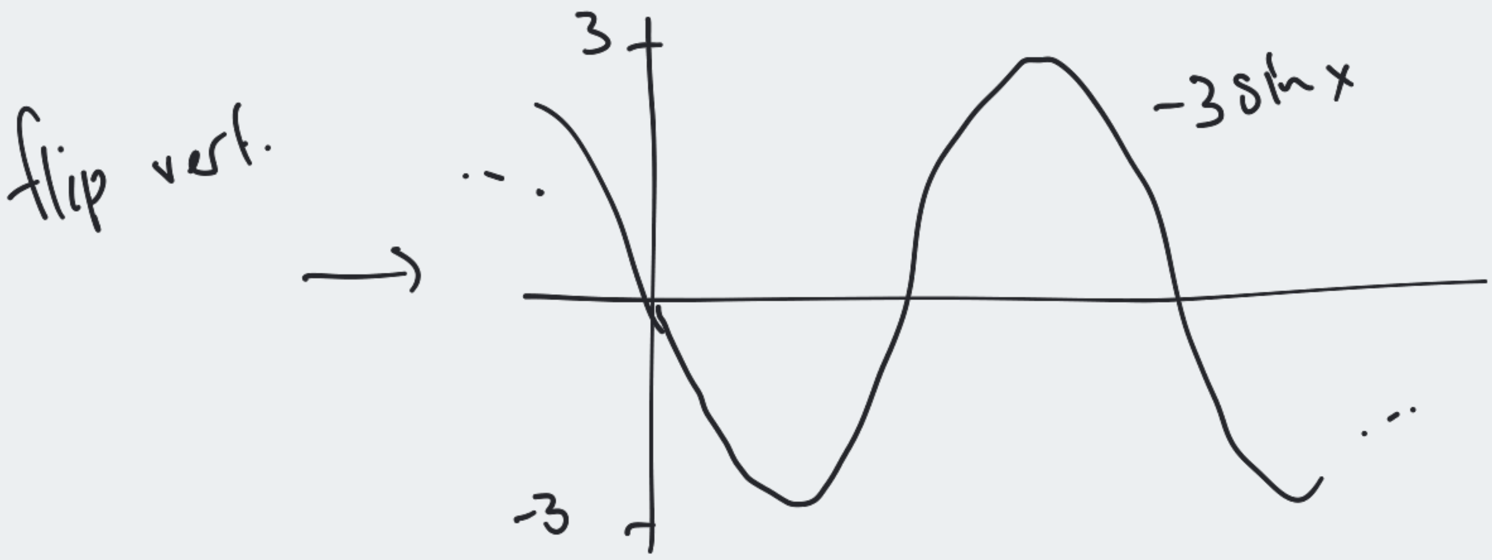
$y = f(x/c)$ ... stretch - - - -

if $c<0$, also flips horizontally.



So $-3\sin x$





You try sketch graphs of

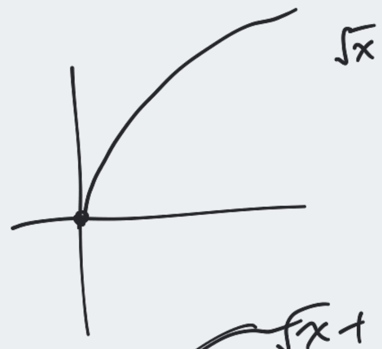
$\sqrt{x} + 3$

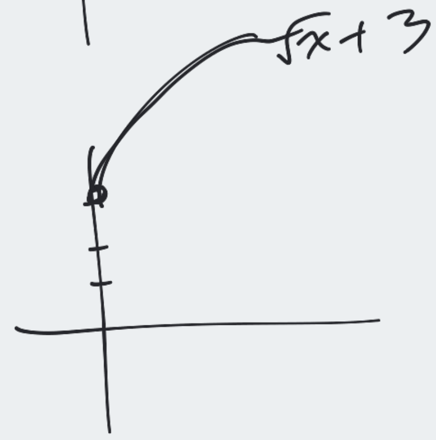
$\sqrt{2x} + 3$

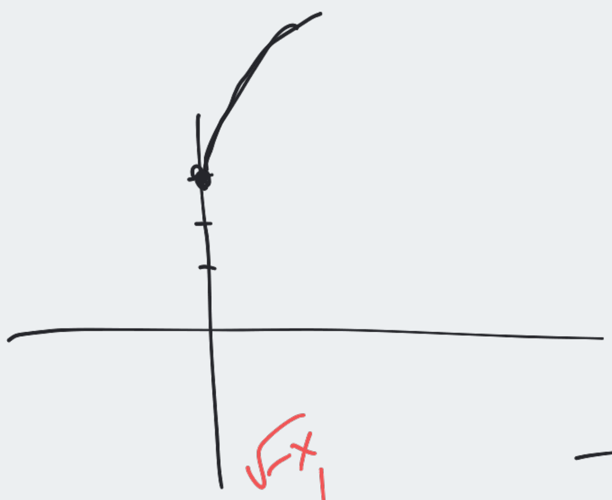
$\sqrt{-x}$

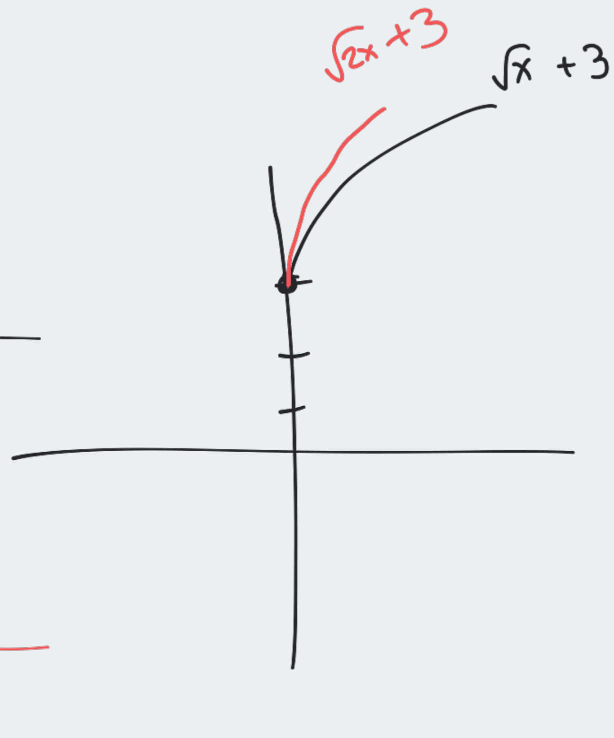
$4\sqrt{x+1}$

$7\sqrt{3+x} + 4$

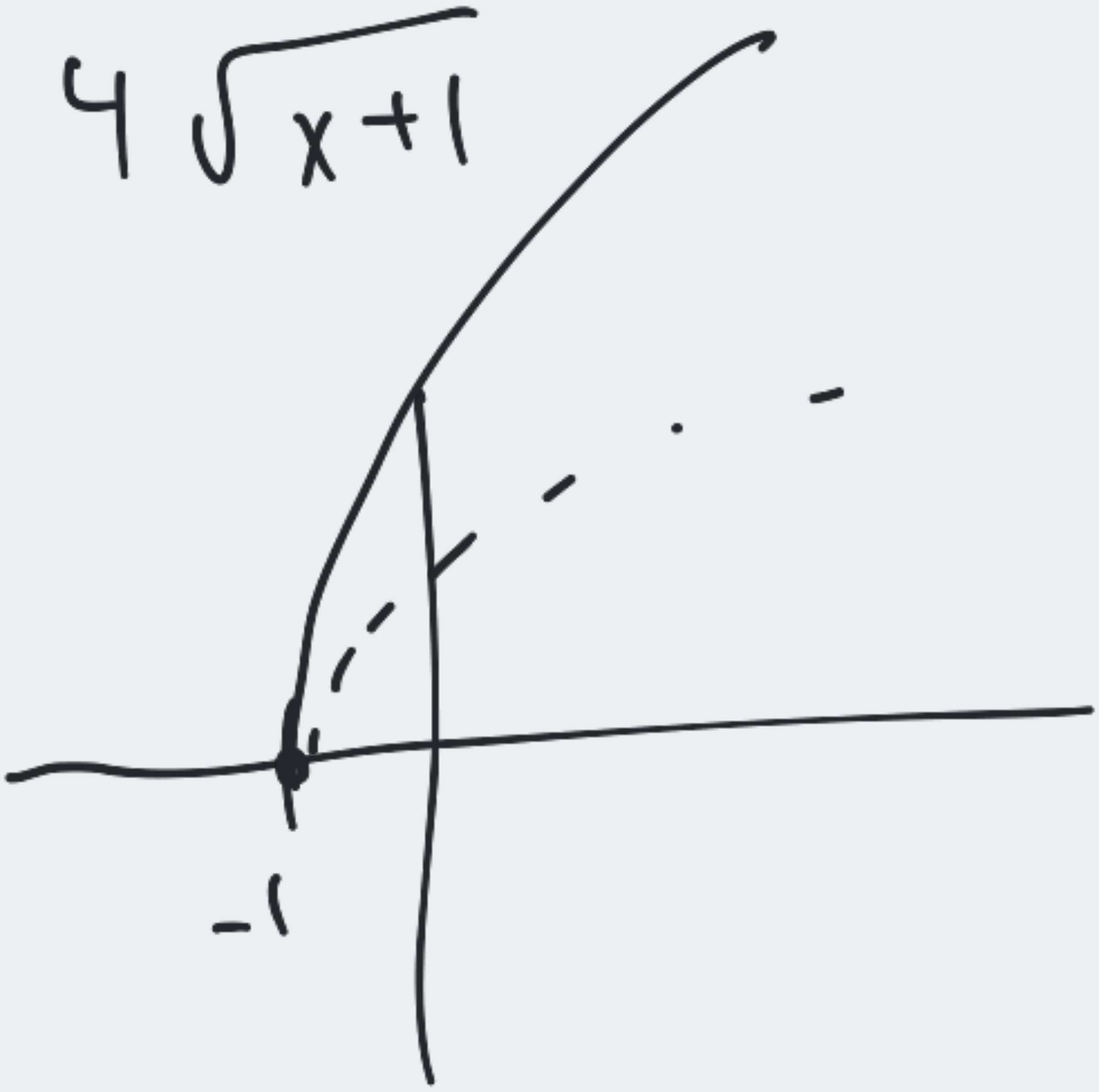




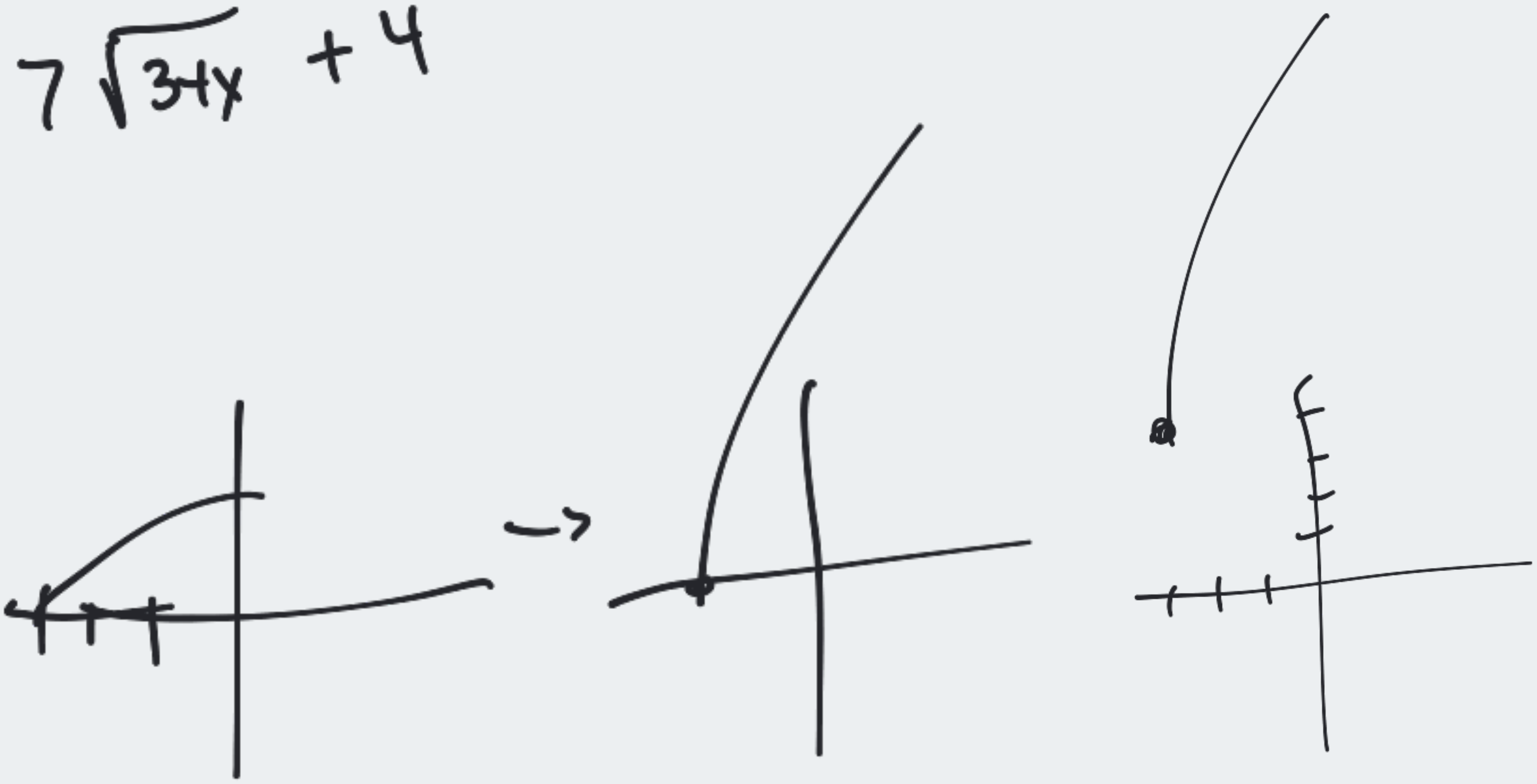




$4\sqrt{x+1}$



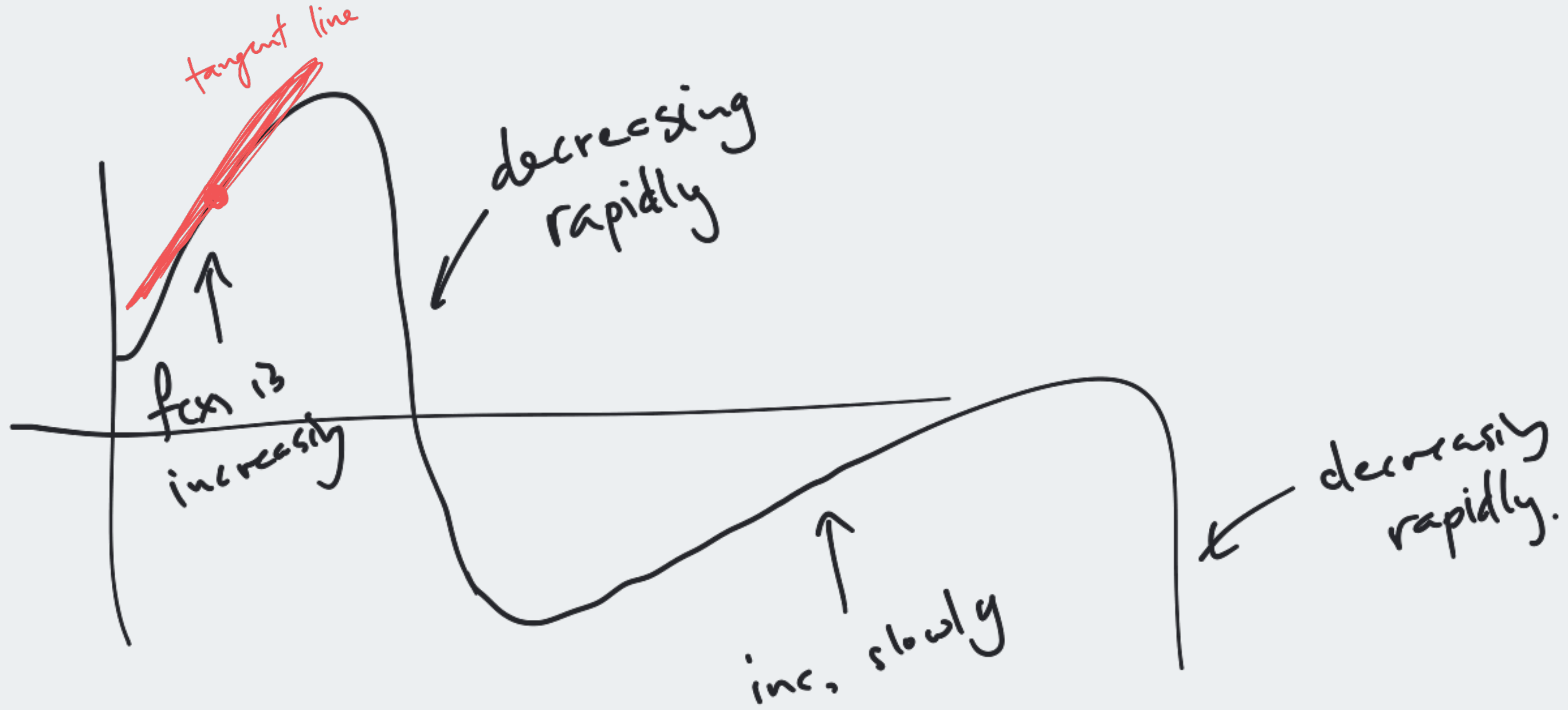
$7\sqrt{3+x} + 4$



Tangents & Velocity

calc. is about change

how fast is f(x) changing?



"changing" has to do with the slope

slope of a line is easy: $\frac{\text{rise}}{\text{run}}$