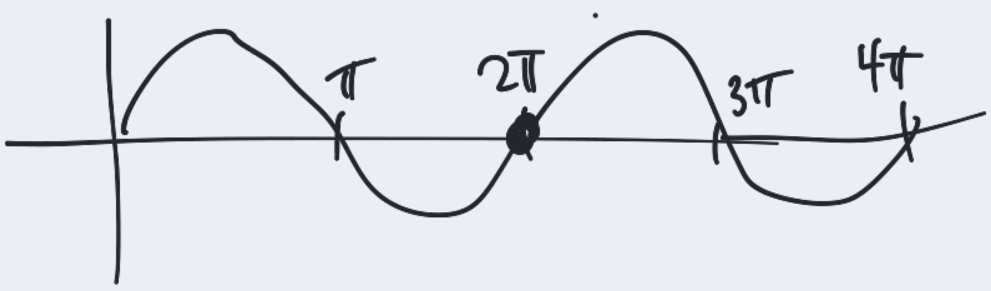
Homework today!

Quiz Friday!

$f(x) = \sin\left(\frac{\pi}{x}\right)$



Find $\lim_{x \to 0} \sin \frac{\pi}{x}$

Choose x-values close to 0:

$x = .1 \to \sin \frac{\pi}{.1} = \sin 10\pi = 0$

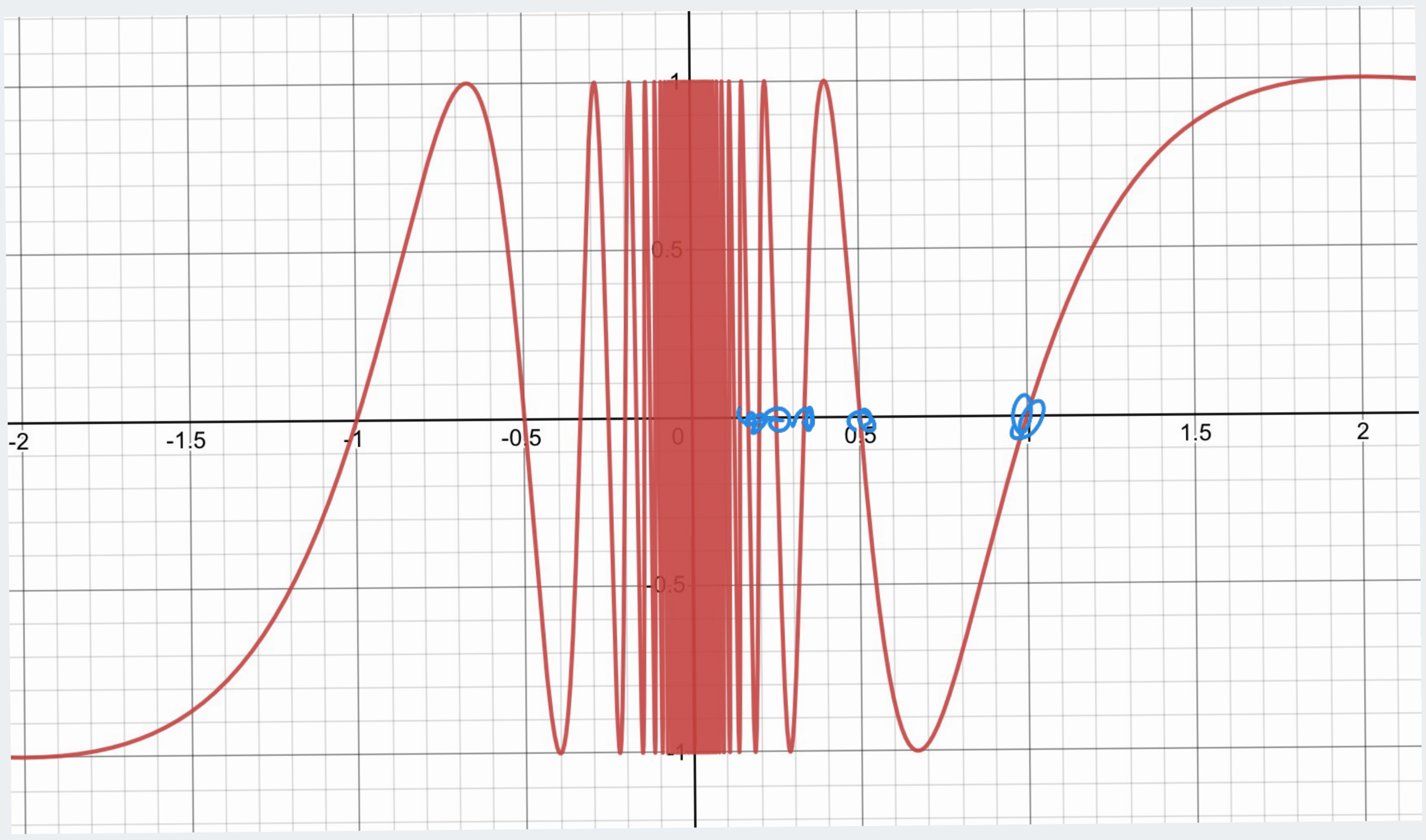
$x = .01 \longrightarrow \sin \frac{\pi}{.01} = \sin 100\pi = 0$

$x = .001 \to \sin \frac{\pi}{.001} = \sin 1000\pi = 0$

seems like $\lim_{x \to 0} \sin \frac{\pi}{x} = 0$

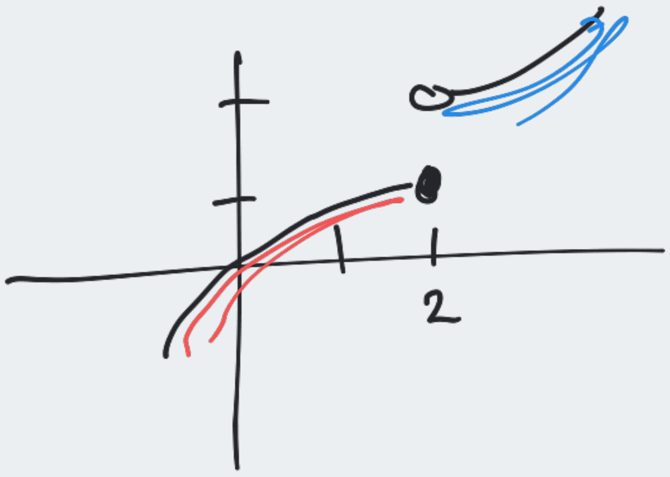
$x = .009 \quad \sin \frac{\pi}{.009} = -.34$ not really close to 0.

The limit is not 0!



$\lim_{x \to 0} \sin \frac{\pi}{x}$ DNE.

One-sided Limits



Here, $\lim_{x \to 2} f(x)$ DNE.

(approaches 2 different values on the 2 sides)

$\lim_{x \to 2^-} f(x) = 1$

limit from the left

$\lim_{x \to 2^+} f(x) = 2$

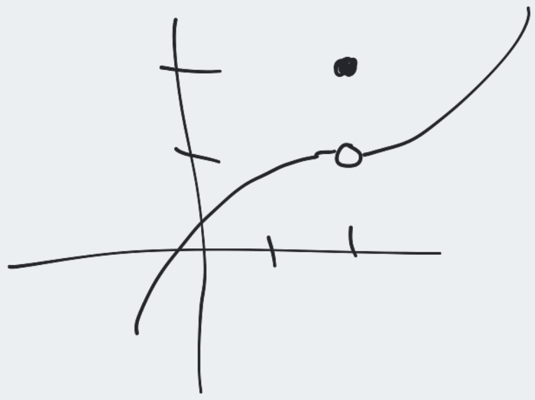
limit from the right.

$$\lim_{x \to a^-} f(x) = L \qquad \lim_{x \to a^+} f(x) = L$$

means:

As x gets really close to a, but $x < a$, then the y value is close to L. . . . . . $x > a$ . . .

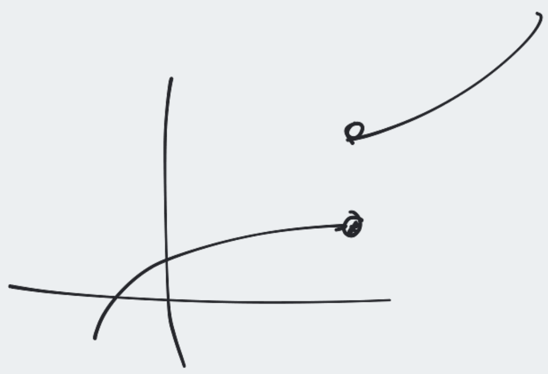
Sometimes they're equal:



$f(2) = 2$

$$\left.\begin{aligned} \lim_{x \to 2^-} f(x) &= 1 \\ \lim_{x \to 2^+} f(x) &= 1 \end{aligned}\right] \text{ so } \lim_{x \to 2} f(x) = 1$$

different looks like



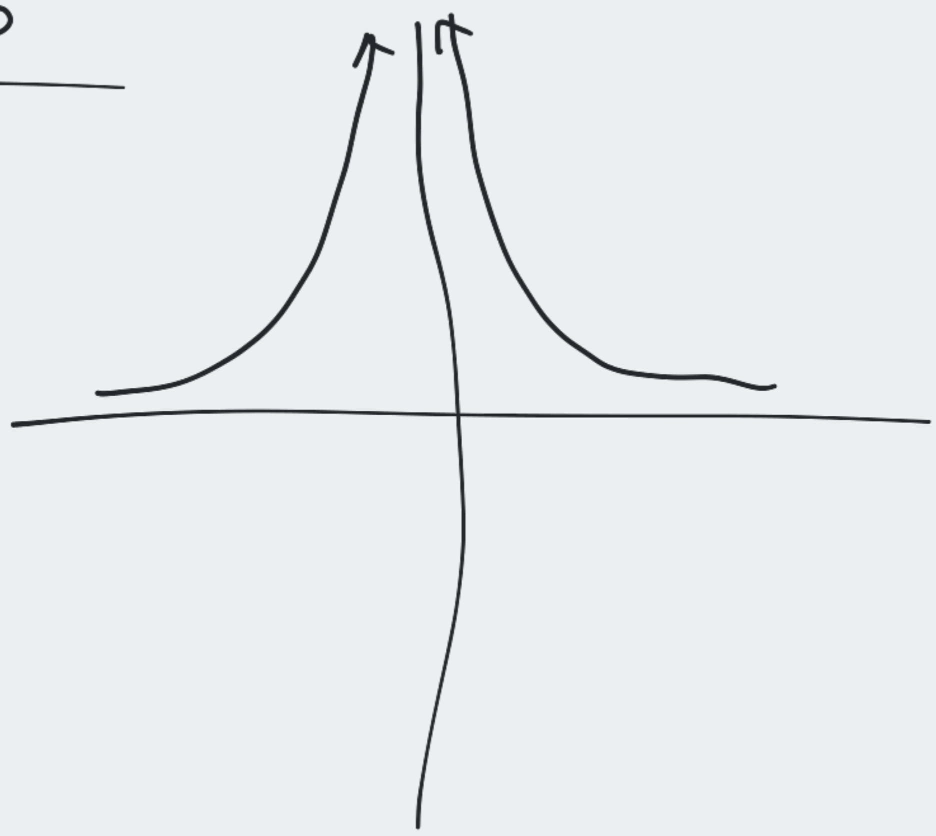
Theorem If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$,

then $\lim_{x \to a} f(x)$ exists, and

$$\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x).$$

Infinite Limits

$\lim_{x \to 0} \frac{1}{x^2}$ DNE



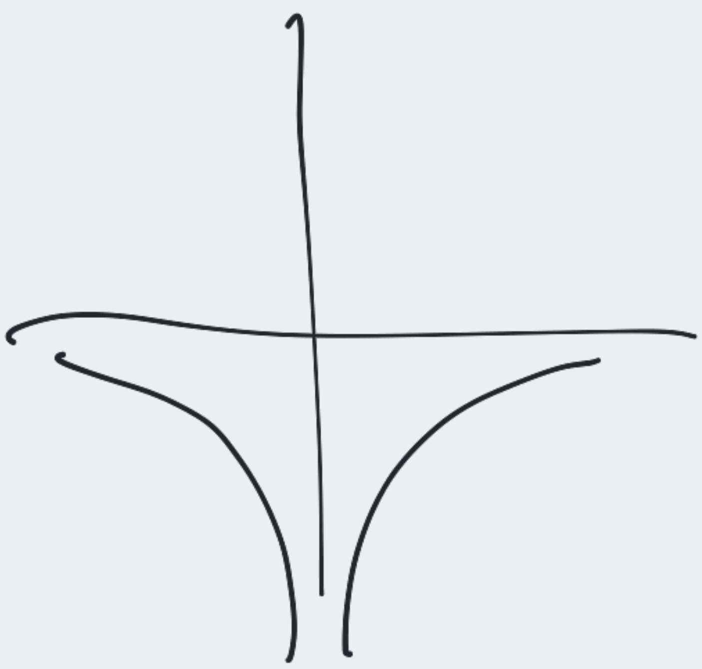
but more specifically,

$\underbrace{\lim_{x \to 0} \frac{1}{x^2} = \infty}$

This means: when x is close to 0, the y values become arbitrarily large.

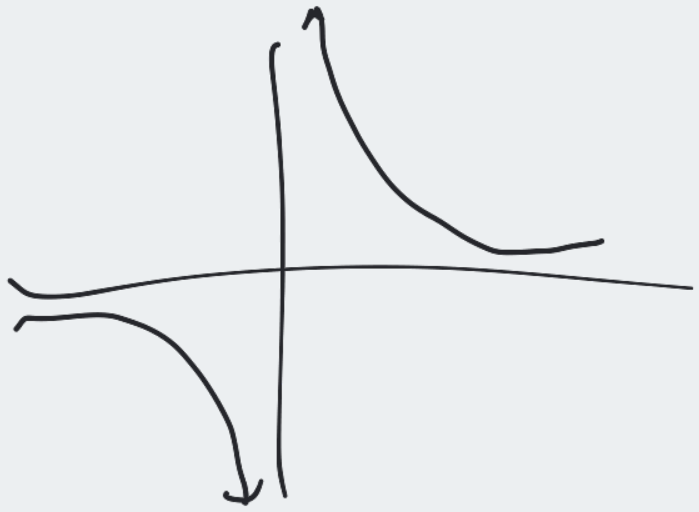
We can also say $\lim_{x \to a} f(x) = -\infty$

means: as x approaches a, the y values become arbitrarily large negative.



$\lim_{x \to 0} \frac{-1}{x^2} = -\infty$

$\frac{1}{x}$:



$\lim_{x \to 0} \frac{1}{x}$ is not ∞ or $-\infty$,

but $\lim_{x \to 0^-} \frac{1}{x} = -\infty$

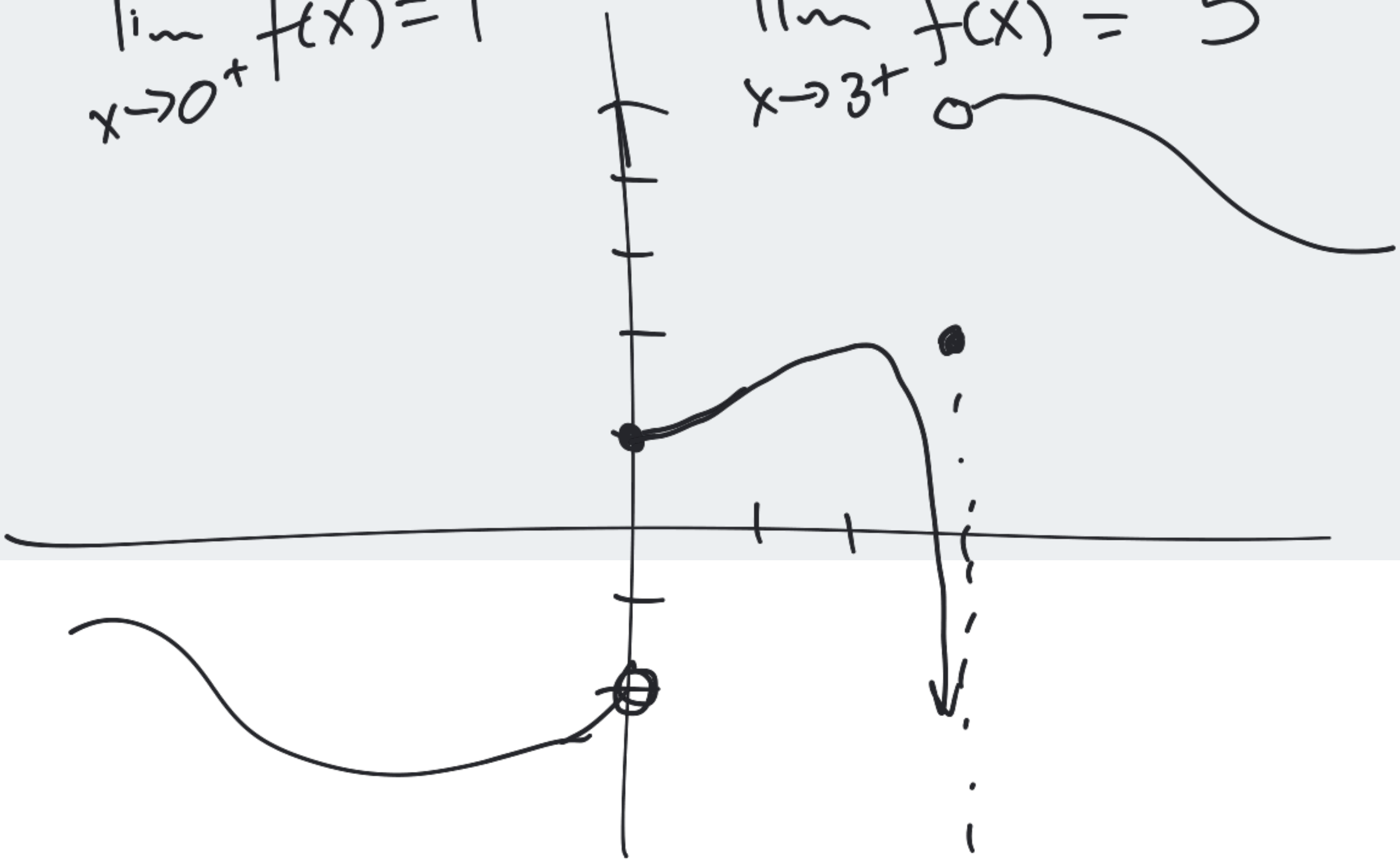
$\lim_{x \to 0^+} \frac{1}{x} = \infty$

You invent a graph with:

$f(0) = 1$ $\qquad$ $f(3) = 2$

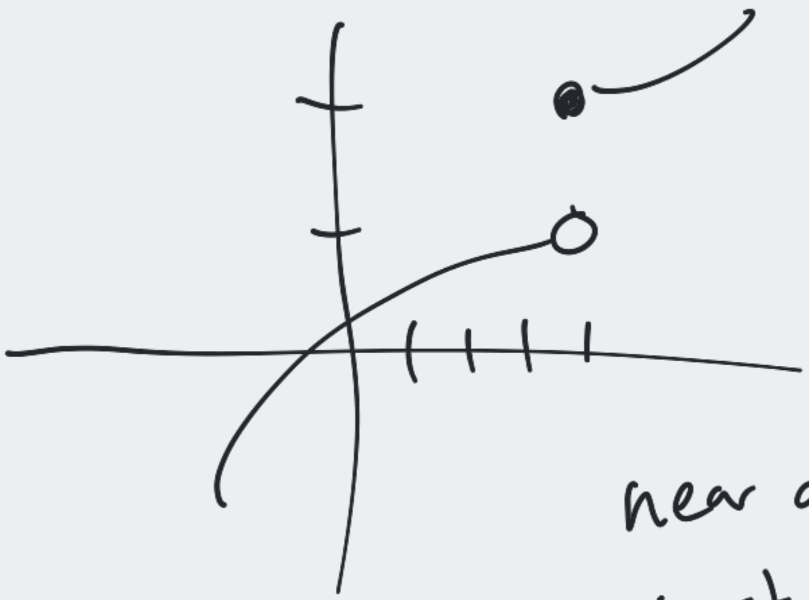
$\lim_{x \to 0^-} f(x) = -2$ $\qquad$ $\lim_{x \to 3^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = 1$ $\qquad$ $\lim_{x \to 3^+} f(x) = 5$



Limit Laws

Usually, limits on graphs are not very interesting.



Near $x=4$, the limit is a bit weird

near all other points, the limit just equals the function value.

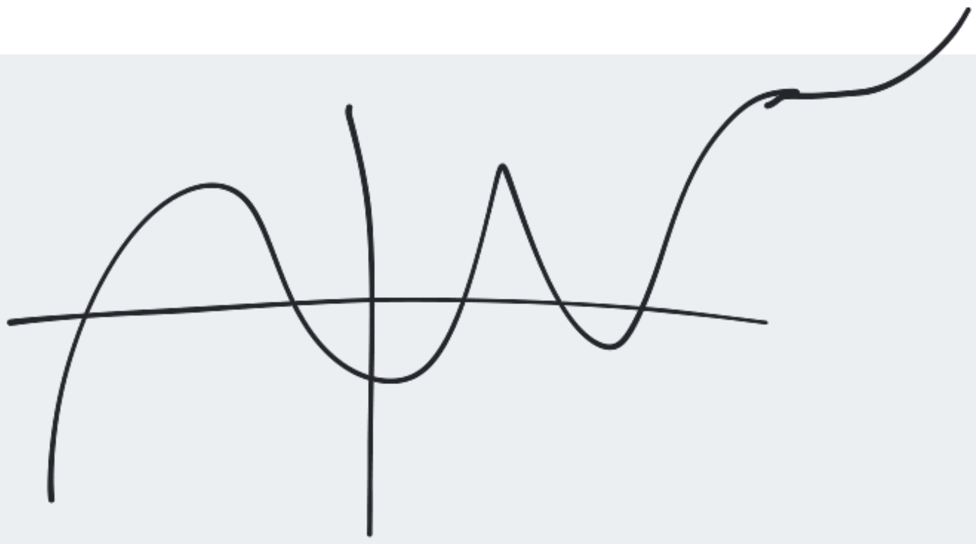
$$f(0) = 1/3$$

$$\lim_{x \to 0} f(x) = 1/3$$

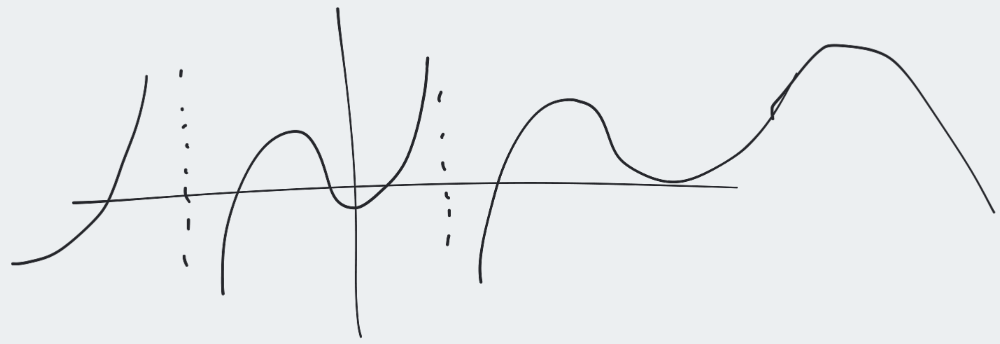
When there's no funny business on the graph, we expect

$$\lim_{x \to a} f(x) = f(a)$$

This is always true of polynomials



also true for any rational function



For any polynomial or rational function,

$$\lim_{x \to a} f(x) = f(a)$$

this is how we find the limit.