Rationalize trick

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \rightarrow \frac{\sqrt{9}-3}{9-9} = \frac{0}{0}$$

go back & simplify

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{\sqrt{x}\sqrt{x}-9}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{\cancel{x-9}}{(\cancel{x-9})(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Try

$$\lim_{x\to 1} \frac{\sqrt{3+x}-2}{x-1} = \lim_{x\to 1} \frac{\sqrt{3+x}-2}{x-1} \cdot \frac{\sqrt{3+x}+2}{\sqrt{3+x}+2}$$

$$= \lim_{x\to 1} \frac{(3+x)-4}{(x-1)(\sqrt{3+x}+2)} = \lim_{x\to 1} \frac{\cancel{x-1}}{(\cancel{x-1})(\sqrt{3+x}+2)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{3+x}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$$

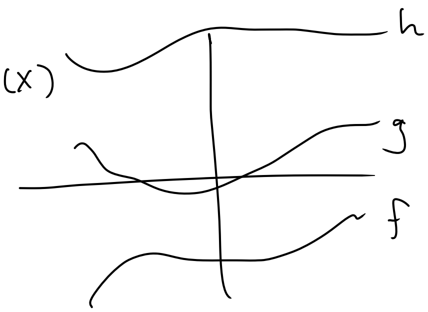
$$\lim_{x\to -1} \frac{x^2-1}{\sqrt{x^2+8}-3} \cdot \frac{\sqrt{x^2+8}+3}{\sqrt{x^2+8}+3}$$

$$= \lim_{x\to -1} \frac{(x^2-1)(\sqrt{x^2+8}+3)}{x^2+8-9} = \lim_{x\to -1} \frac{(\cancel{x^2-1})(\sqrt{x^2+8}+3)}{\cancel{x^2-1}}$$

$$= \lim_{x \to -1} \sqrt{x^2+8} + 3 = \sqrt{9} + 3 = 6$$

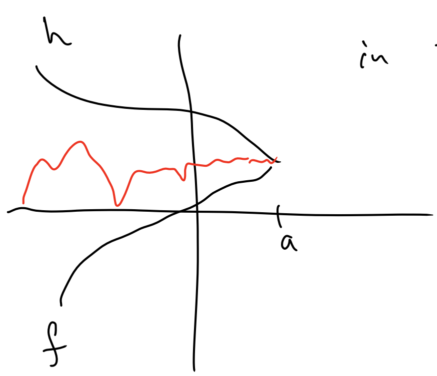
The Squeeze Theorem

<u>If</u>: $f(x) \leq g(x) \leq h(x)$



"squeeze" is when

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x),$$



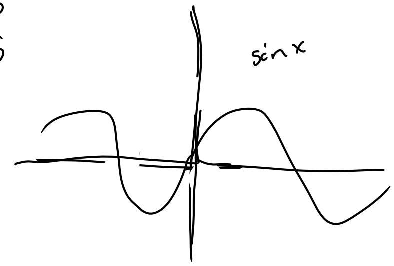
in that case, g must have the same limit.

<u>Thm</u> <u>If</u> $f(x) \leq g(x) \leq h(x)$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x)$,

then $\lim_{x \to a} g(x) = \lim_{x \to a} f(x) = \lim_{x \to a} h(x)$.

We can use this to find weird limits by squeezing the function

Ex 1 $\lim_{x \to 0} \frac{\sin x}{x} \to \frac{\sin 0}{0} = \frac{0}{0}$



Let's squeeze it!

$$\cos x \le \frac{\sin x}{x} \le 1$$

but $\lim_{x \to 0} \cos x = \cos 0 = 1$

$\lim_{x \to 0} 1 = 1$

Two outer limits are both 1,
so the inside is also 1.

So $\lim_{x \to 0} \frac{\sin x}{x} = 1$

What a limit really means

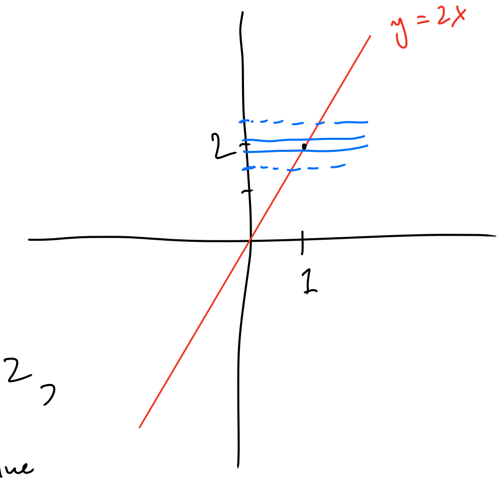
$$\lim_{x \to a} f(x) = L$$

When x is really really close to a,
then f(x) is really close to L

When x approaches a, ...

Not a real definition!

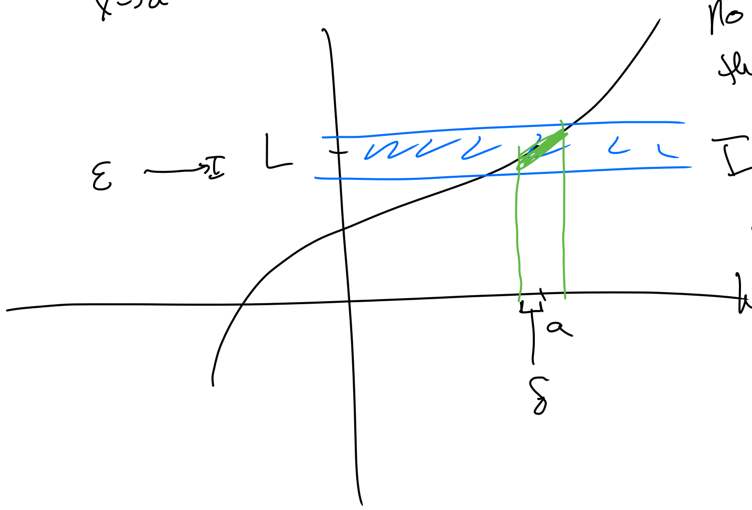
$$\lim_{x \to 1} 2x = 2$$



The y-values will get as close as you want to 2, provided that the x value is close enough to 1.

However close you want the y to be to 2, there is some distance around x where this happens.

$$\lim_{x \to a} f(x) = L$$



No matter how small the blue window, I can make a green window which lands inside the blue.

The True Definition:

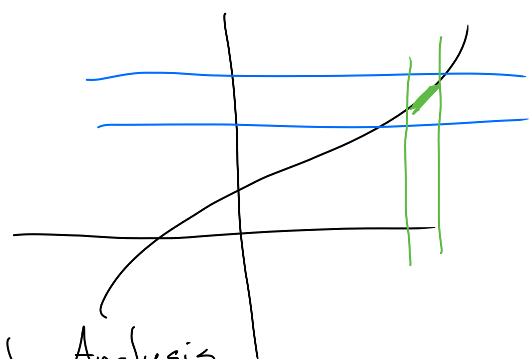
For any $\varepsilon > 0$, there exists some $\delta > 0$ such that

if $|x-a| < \delta$, then $|f(x) - L| < \varepsilon$

if x is inside the green

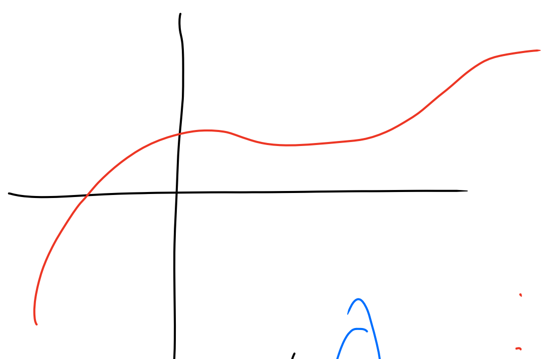
y is inside the blue.

discovered in 1800s



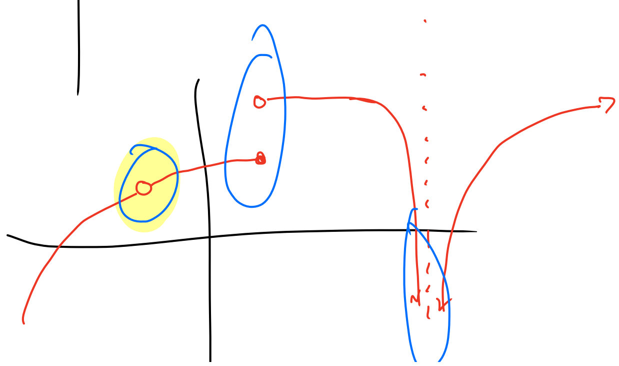
More on this in Math 3371 - Real Analysis

Continuity



is continuous

it's all nice - like



not continuous because of blue spots

For the kids: Continuous means you can draw the graph without lifting your pencil.

Real definition: $f(x)$ is continuous at some point $x = a$

means: $$\lim_{x \to a} f(x) = f(a)$$

i.e.

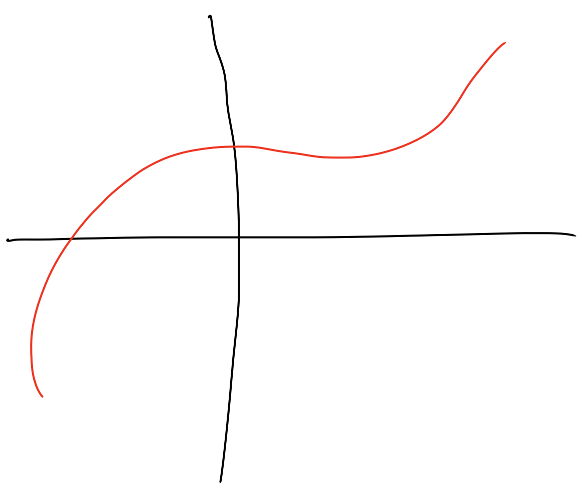
- $f(a)$ exists

and

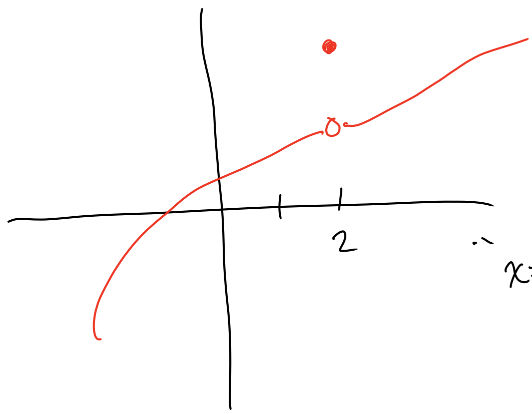
- $\lim_{x \to a} f(x)$ exists

and

- $\lim_{x \to a} f(x) = f(a)$



This is continuous for all real #s.



This is continuous at all real #s except 2.

∴ "$x=2$ is a discontinuity point"