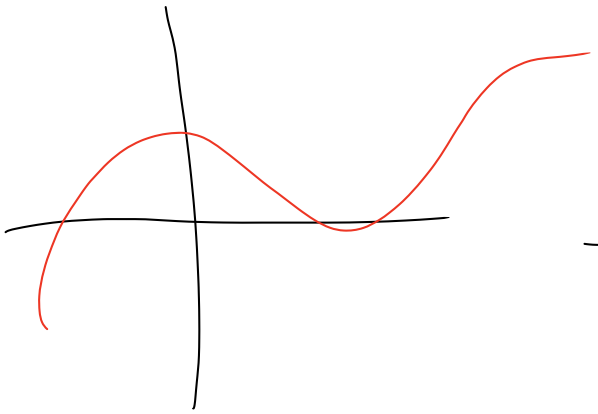
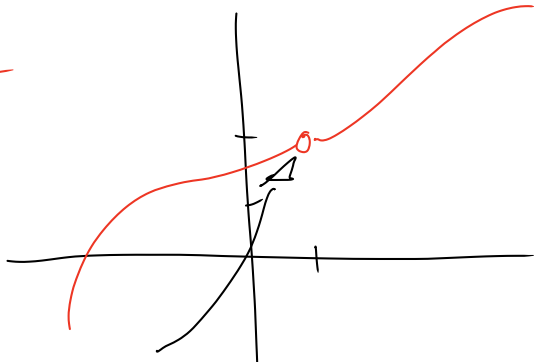
Continuity

Continuous at $x=a$ means

$$\lim_{x \to a} f(x) = f(a)$$





$x=1$ is a discontinuity point.

Finding discontinuities

Ex) $f(x) = \dfrac{x^2 - 2x - 3}{x - 1}$ Find any discontinuities

need the denominator to be nonzero.

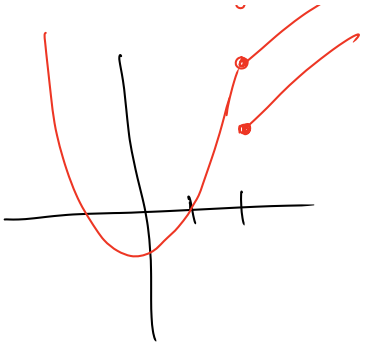
denom is zero when $x = 1$.

So $f(x)$ is continuous for all real #s except 1.

$x=1$ is a discontinuity.

one of these lines

Ex1 $f(x) = \begin{cases} 3x^2 - 1 & \text{if } x < 2 \\ x + 10 & \text{if } x \geq 2 \end{cases}$



Need to check if the pieces meet up.

Plug $x = 2$ for each one:

$3 \cdot 2^2 - 1 = 11$

$2 + 10 = 12$ ← different, so f is not continuous at $x = 2$.

If there's no denominators or pieces, it will be continuous on its domain

e.g polynomials, sin & cos are all continuous everywhere.

Give intervals where $f(x) = \dfrac{x+5}{x^2+4x-5}$ is continuous.

check when denom = 0:

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x + 5 = 0 \qquad x - 1 = 0$

$x = -5 \qquad x = 1$

it is continuous on the intervals:

$(-\infty, -5)$, $(-5, 1)$, $(1, \infty)$

$(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$

Ex/ Give intervals where it's continuous:

$$\frac{\sqrt{x+3}}{x-10}$$

denom must be nonzero,
inside of root must be positive.

denom $= 0$ when $x = 10$, so x cannot be 10.

also, need $x + 3 \geq 0$,

so $x \geq -3$.

Continuous for every $x \geq -3$ except $x = 10$.

as intervals: $[-3, 10) \cup (10, \infty)$

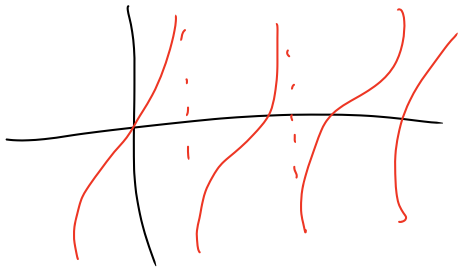
Rules for continuous functions

Thm If f & g are continuous, then:

- $f + g$ is continuous
- $f - g$ is continuous
- $f \cdot g$ is continuous

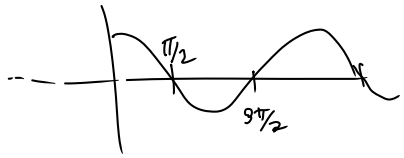
[$x^2 + 7x + \sin x$ is continuous.]

- cf for any real # c.
- f/g is continuous whenever $g \neq 0$.
- $f(g(x))$ is continuous $\sin(x^2+3)$ $\frac{x^2-1}{x+3}$



Ex/ Where are discontinuities of $\tan x \cdot \left(\frac{x-1}{x+2}\right)$

discont. of $\frac{x-1}{x+2}$ are: $x=-2$



discont of $\tan x = \frac{\sin x}{\cos x}$ discont whenever $\cos x = 0$

i.e. $\pi/2 + \pi n$ for any integer n.

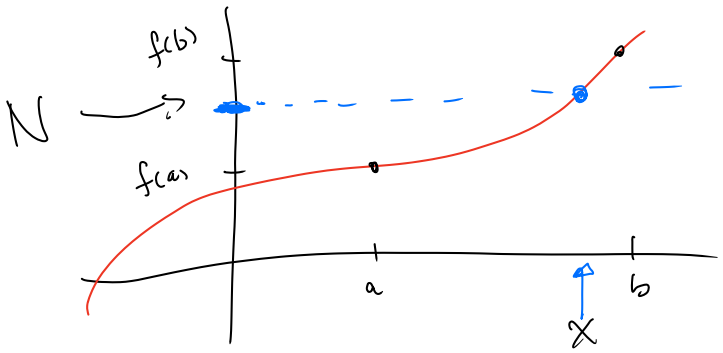
The discontinuities are

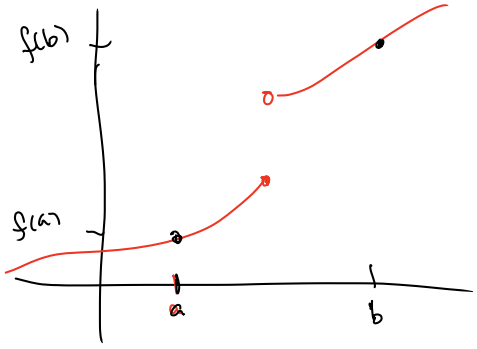
-2, and $\pi/2 + \pi n$.

Intermediate Value Theorem

If a cont. function goes thru 2 y-values, then it also hits all y-values in between.

↑ "intermediate values"





Thm Let $f(x)$ be continuous on $[a,b]$ and let N be any number between $f(a)$ & $f(b)$,

Then there is some x with

$$a \le x \le b$$

with $f(x) = N$.

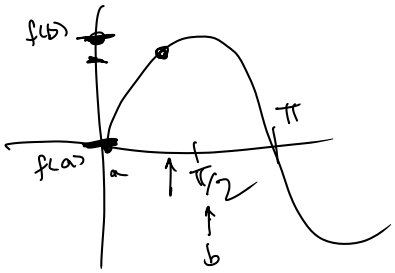
Ex1 Show that there is some x with $\sin x = 0.8675309$

$\sin x$ is continuous, so we can use IVT.

need to say: .8675309 is in between two other values of $\sin x$.

$\sin(0) = 0$

$\sin(\pi/2) = 1$



Since .8675309 is between 0 & 1,

there is some x-value in $(0, \pi/2)$

with $\sin x = .8675309$.