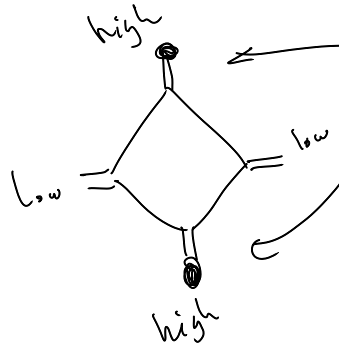
IVT & Wobbly Table

Square table, even legs,
wobbles because the ground is uneven.

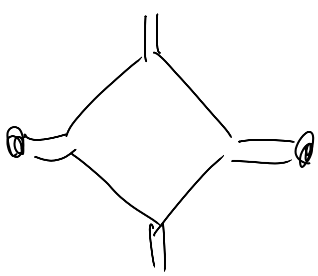
Before



touching
others are floating.

rotate $90°$,
shove it down so the
same ones touch the ground;

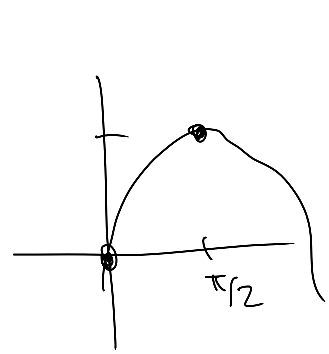
After



others will be buried.

Show there is some x with

$$\sin x = .8675309$$



IVT can demonstrate solutions to equations.

Show that $x^3 + 5x - 100$ has a root.

some x with $f(x) = 0$.

Find some x with $f(x) > 0$,

" " " " $f(x) < 0$.

for $x = 100$, $100^3 + 5 \cdot 100 - 100$ is positive.

for $x = 2$ $2^3 + 5 \cdot 2 - 100$ is negative.

So $f(x) = 0$ for some x in $(2, 100)$

if I plug $x = 5$:

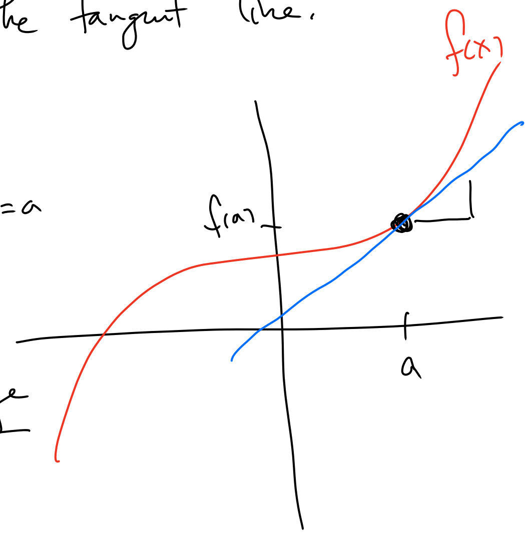
$5^3 + 5 \cdot 5 - 100$

$125 + 25 - 100 = 50$

so $f(x) = 0$ for x in $(2, 5)$

The Derivative

The fundamental measure of how fast a function is changing at a point, it is the slope of the tangent line.

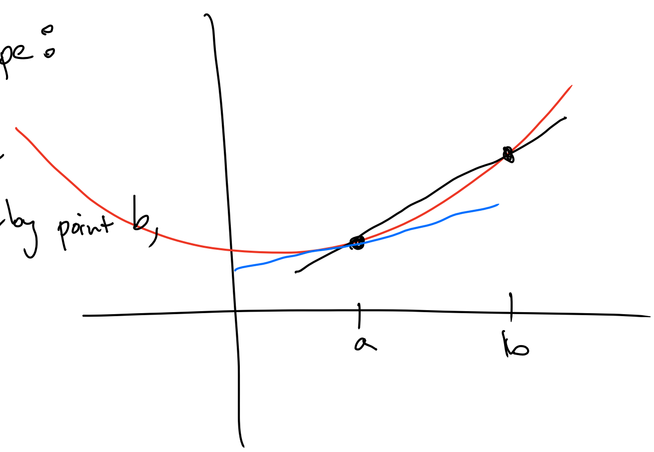


Different names:

- slope of the tangent line at $x=a$
- derivative of $f(x)$ at $x=a$
- the instantaneous rate of change at $x=a$
- $f'(a)$

To find the slope:

Measure the average slope to a nearby point b, then do $\lim_{b \to a}$



$$\boxed{f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a}}$$

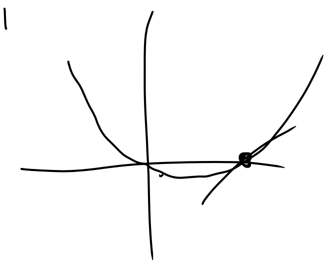
Ex1 find the slope of $f(x) = x^2 - x$ at $a = 1$,

use $a = 1$:

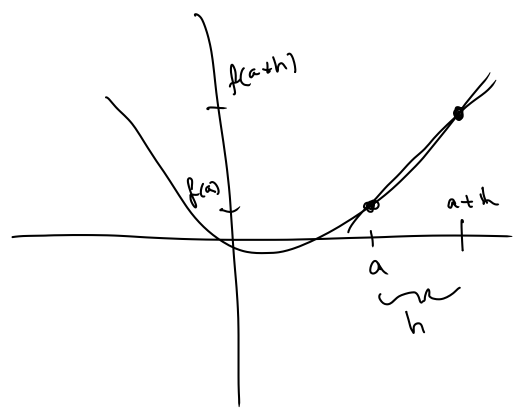
$$\lim_{b \to 1} \frac{f(b) - f(1)}{b - 1} = \lim_{b \to 1} \frac{b^2 - b - (1^2 - 1)}{b - 1}$$

$$= \lim_{b \to 1} \frac{b^2 - b}{b - 1} = \lim_{b \to 1} \frac{b\cancel{(b-1)}}{\cancel{b-1}}$$

$$= \lim_{b \to 1} b = 1.$$



Another version:



$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

$f(x) = x^2 - x$ slope at $a = 1$.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

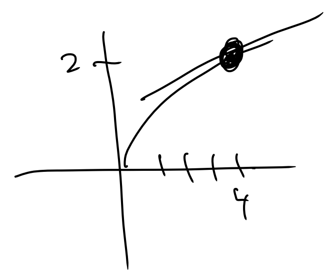
$$= \lim_{h \to 0} \frac{(1+h)^2 - (1+h) - (1^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1}+2h+h^2-\cancel{1}-h}{h}$$

$$= \lim_{h \to 0} \frac{h+h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(1+h)}{\cancel{h}}$$

$$= 1+0 = 1$$

Find the equation of the tangent line for $y=\sqrt{x}$ at $x=4$



Use the point-slope form:

$$y - \underset{\overset{\uparrow}{2}}{y_0} = m(x - \underset{\overset{\uparrow}{4}}{x_0})$$

<u>The m:</u>

$$f'(4) = \lim_{h \to 0} \frac{f(4+h)-f(4)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h}-\sqrt{4}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$= \lim_{h \to 0} \frac{(\cancel{4}+h)-\cancel{4}}{h(\sqrt{4+h}+2)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h} + 2}$$

$$= \frac{1}{\sqrt{4} + 2} = \frac{1}{2+2} = \frac{1}{4}$$

$$y - y_0 = m(x - x_0)$$

$$y - 2 = \frac{1}{4}(x - 4)$$

$$y - 2 = \frac{1}{4}x - 1$$

$$\boxed{y = \frac{1}{4}x + 1}$$