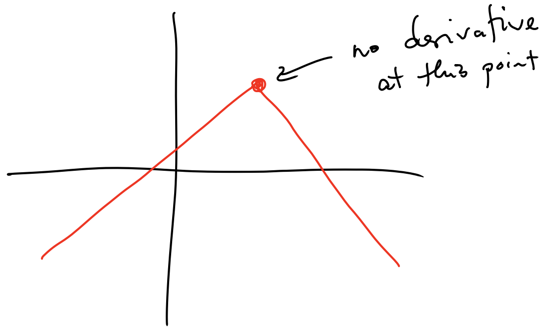
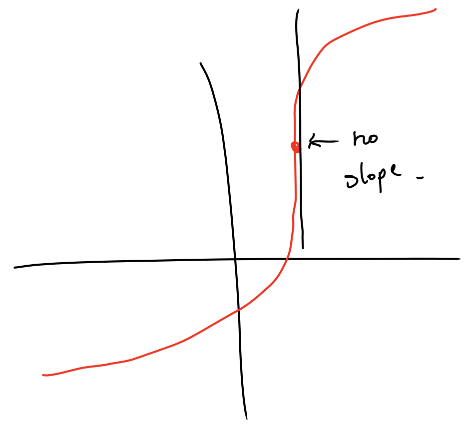
Some functions are not differentiable at certain points

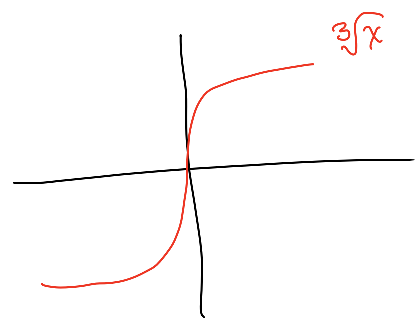




Some continuous functions have non-differentiable points

<u>Ex)</u> $y = \sqrt[3]{x}$

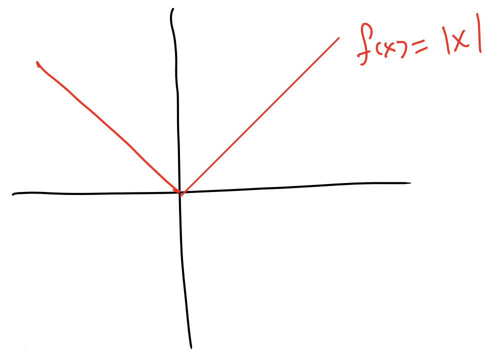
$f(x) = \sqrt[3]{x}$



Turns out: $f'(x) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$

notice: $f'(0) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{0^2}} = \frac{1}{0}$ DNE

Pointy:



$$f(x) = |x|$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{|x+h| - |x|}{h}$$

so $f'(0)$:

$$f'(0) = \lim_{h\to 0} \frac{|h| - |0|}{h} = \lim_{h\to 0} \frac{|h|}{h}$$

$$= \lim_{h\to 0} \begin{cases} h/h & \text{if } h > 0 \\ -h/h & \text{if } h \leq 0 \end{cases}$$

$$= \lim_{h\to 0} \begin{cases} 1 & \text{if } h > 0 \\ -1 & \text{if } h \leq 0 \end{cases}$$

lim DNE because from left you get -1,
from right $+1$.

So $f'(0)$ DNE.

True fact: Any differentiable function must be continuous.

Proof We'll assume f is differentiable at a.

we'll show $\underbrace{\lim_{x \to a} f(x) = f(a)}$ ← def. of continuous.

use $\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$

Simple tricks:

$$\frac{f(x) - f(a)}{x - a} \cdot (x - a) = f(x) - f(a)$$

take $\lim_{x \to a}$ on both sides:

$$\underbrace{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}} \cdot \underbrace{\lim_{x \to a} (x - a)} = \lim_{x \to a} \left(f(x) - f(a) \right)$$

↑

$$\left(f'(a) \right) \cdot (a - a) = \left(\lim_{x \to a} f(x) \right) - f(a)$$

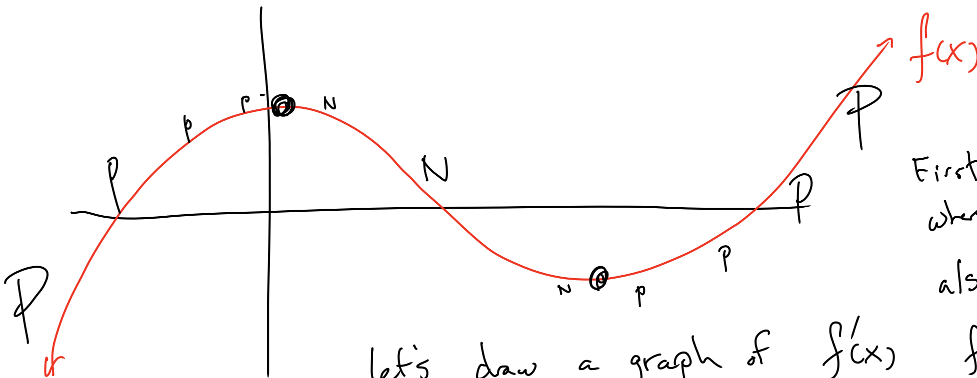
$$0 = \lim_{x \to a} f(x) - f(a)$$

$$\underline{f(a) = \lim_{x \to a} f(x)}$$

QED

Shown

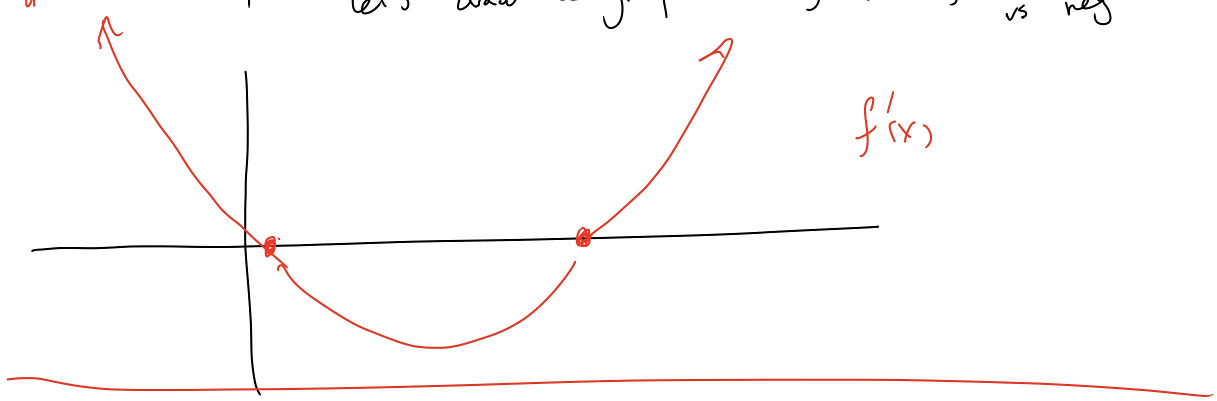
Derivatives on Graphs

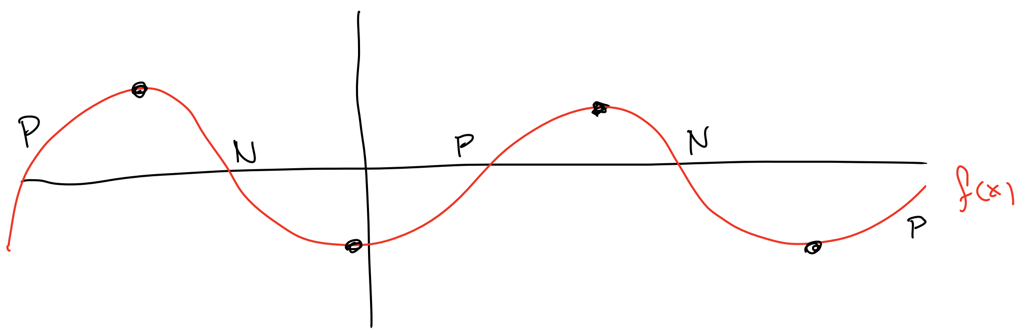


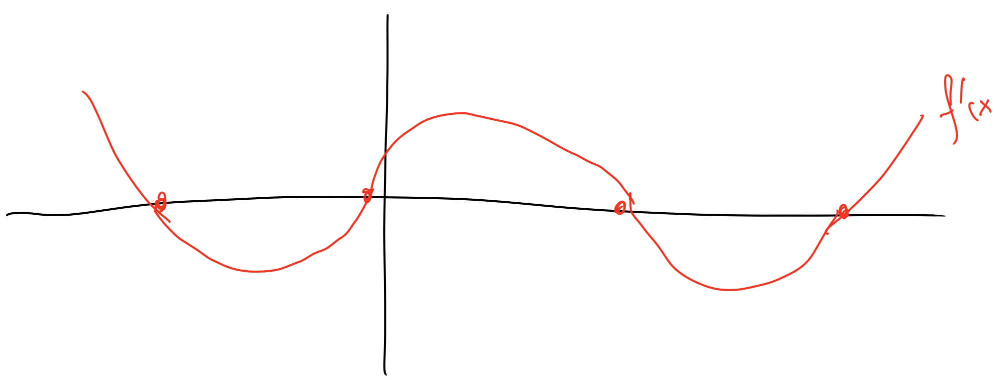
First identify when $f'(x) = 0$

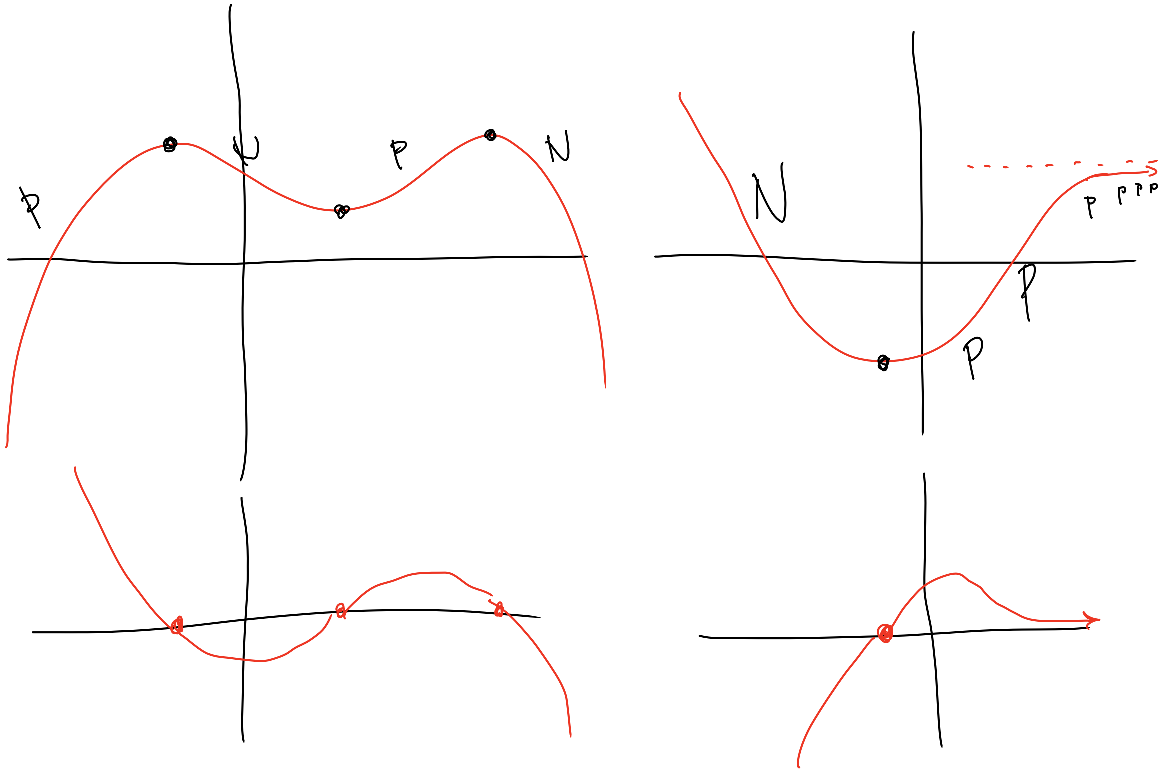
also say when f' is pos vs neg.

let's draw a graph of $f'(x)$









Various notations for the derivative.

for $y = f(x)$

we write the deriv. as:

$$f'(x) \quad \text{or} \quad \frac{dy}{dx} \quad \text{or} \quad \frac{d}{dx} f(x)$$

not really a fraction, meant to resemble $\frac{\text{rise}}{\text{run}}$

the derivative of

$\frac{d}{dx} x^2 = 2x$ means

the deriv. of x^2 is $2x$.