Higher Derivatives

We can take the derivative of the derivative!

written $f''(x)$ deriv. of $f'(x)$

"second derivative" "first derivative"

Ex| $f(x) = 5 + 2x - x^2$ find $f''(x)$.

from last time:

$$f'(x) = 2 - 2x$$

So, $f''(x)$ is the deriv. of

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 - 2(x+h) - (2 - 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{2x} - 2h - \cancel{2} + \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{h} = -2$$

If $s(t)$ is the position of an object,

then $s'(t)$ represents: velocity

then $s''(t)$ is acceleration.

then $s'''(t)$ is jerk

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f', f'', f''', $f^{(4)}$, $f^{(5)}$, etc.

$\frac{d}{dx} f$, $\frac{d^2}{dx^2} f$, $\frac{d^3}{dx^3} f$

$\frac{d}{dx} \frac{d}{dx} f$

$\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, $\frac{d^3y}{dx^3}$, etc.

Tricks!

Finding derivs of common functions can be very easy if you know some basic tricks.

including: polynomials, trig functions, radicals
any product, quotient, or composition of these.

Constant functions

$$f(x) = c$$

then $\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c - c}{h} = 0$

deriv of a constant is 0.

$$\boxed{\frac{d}{dx} c = 0}$$

Power of x

$f(x) = x^n$ n is a positive whole number

warm up: x^4

$$\frac{d}{dx} x^4 = \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0}$$

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$\underline{(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4}$$

Pascal's triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$\lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - \cancel{x^4}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3)}{\cancel{h}} = 4x^3 + 6x^2\cdot 0 + 4x\cdot 0^2 + 0^3$$

$$= 4x^3$$

$$\frac{d}{dx}x^4 = 4x^3$$

P's Δ:

row n: 1 n ??? n 1

$$\frac{d}{dx}x^n = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + \dots + h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(nx^{n-1} + \dots + h^{n-1})}{\cancel{h}}$$

all these have extra factors of h.

$$= \lim_{h\to 0} nx^{n-1} + (\dots\dots) = nx^{n-1}$$

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

the power rule.

$$\frac{d}{dx} x^6 = 6x^5 \qquad \frac{d}{dx} x^{1000} = 1000\, x^{999}$$

$$\frac{d}{dx} x^4 = 4x^3$$

important special case

$$\frac{d}{dx} x = \frac{d}{dx} x^1 = 1x^0 = 1$$

$$\boxed{\frac{d}{dx} x = 1}$$

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coefficients:

$$\frac{d}{dx}\left(c f(x) \right) = \text{??}\ \frac{d}{dx} f(x)\ \text{??}$$

$$\downarrow$$

$$\lim_{h \to 0} \frac{c f(x+h) - c f(x)}{h} = \lim_{h \to 0} \left(c \cdot \frac{f(x+h) - f(x)}{h} \right)$$

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = c \cdot \frac{d}{dx} f(x)$$

$$\boxed{\frac{d}{dx}\left(c\, f(x)\right) = c\,\frac{d}{dx} f(x)}$$

$$\frac{d}{dx} 8x^4 = 8\frac{d}{dx}x^4 = 8 \cdot 4x^3 = 32x^3$$

$$\frac{d}{dx} 8x^4 = 32x^3$$

Now - we can do anything like

$2x^7$, $-3x^4$, etc,

We want $2x^7 + 3x^4$

Addition & Subtraction

$$\frac{d}{dx}\left(f(x) + g(x)\right)$$

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \lim_{h\to 0} \frac{f(x+h)+g(x+h) - (f(x)+g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h} \right)$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

$$\boxed{\begin{aligned} \frac{d}{dx}\left(f(x) + g(x)\right) &= \frac{d}{dx} f(x) + \frac{d}{dx} g(x). \\ \frac{d}{dx}\left(f(x) - g(x)\right) &= \frac{d}{dx} f(x) - \frac{d}{dx} g(x). \end{aligned}}$$

So:

$$\frac{d}{dx}\left(5x^3 - 7x^8 + 2x - 5\right)$$

$$= \frac{d}{dx}(5x^3) - \frac{d}{dx}(7x^8) - \frac{d}{dx}(2x) - \frac{d}{dx}(5)$$

$$= 15x^2 - 56x^7 - 2 - 0$$

$$= 15x^2 - 56x^7 - 2$$