$$\frac{d}{dx}\left(5x^2 - 7x + 8x^4\right)$$

$$= 10x - 7 + 32x^3$$

Say a particle is moving, position is:

$s(t) = 3t^3 - 7t^2 + 5t + 2$ in m,

t in sec.

a) How fast is it moving at $t = 1$ s?

b) What's the acceleration at $t = 2$ s?

a

$$s'(t) = 9t^2 - 14t + 5$$

$$s'(1) = 9 \cdot 1^2 - 14 \cdot 1 + 5 = 0 \text{ m/s}$$

b

$$s''(t) = 18t - 14$$

so $s''(2) = 18 \cdot 2 - 14 = 22 \text{ m/s}^2$

You try:

take deriv:

$$\frac{d}{dx}\left(5x^2 + 7x - 3x^5\right) = 10x + 7 - 15x^4$$

$$6x^{10} - 8 \longrightarrow 60x^9$$

$$\frac{7x^4 - 8x}{x} = \frac{\cancel{x}(7x^3 - 8)}{\cancel{x}} = 7x^3 - 8$$

deriv is: $21x^2$

$$(3x+5)^2 = (3x+5)(3x+5)$$

$$= 9x^2 + 30x + 25$$

deriv $= 18x + 30$

Find any x-values where the slope is horizontal for

$$f(x) = x^4 - 6x^2 + 4$$

$$f'(x) = 4x^3 - 12x$$

$$4x^3 - 12x = 0$$

$$4x(x^2 - 3) = 0$$

$4x = 0$

$\boxed{x = 0}$

$x^2 - 3 = 0$

$x^2 = 3$

$\boxed{x = \pm\sqrt{3}}$

Deriv. of a product or quotient

we know $\frac{d}{dx}(f+g) = \frac{d}{dx}f + \frac{d}{dx}g$

what about $\frac{d}{dx}(f \cdot g)$?

it's <u>not</u> $f' \cdot g'$

check it:

$$\frac{d}{dx}(x^2 \cdot x^5) = \frac{d}{dx}x^7 = 7x^6$$

not the same.

but $\frac{d}{dx}x^2 \cdot \frac{d}{dx}x^5 = 2x \cdot 5x^4 = 10x^5$

We need to work it out:

$$\frac{d}{dx}(f(x) \cdot g(x)) = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} + \frac{f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} f(x+h) \frac{g(x+h) - g(x)}{h} + g(x) \frac{f(x+h) - f(x)}{h}$$

$$= \boxed{f(x) \cdot g'(x) + g(x) f'(x)}$$

The product rule:

$$\frac{d}{dx}\left(\underset{1^{st}}{f(x)} \cdot \underset{2^{nd}}{g(x)}\right) = \underbrace{f(x) g'(x) + g(x) f'(x)}$$

"first thing times deriv of 2^{nd}, plus 2^{nd} times deriv. of 1^{st}"

$$\frac{d}{dx}(x^2 \cdot x^5) = \frac{d}{dx} x^7 = 7x^6$$

$$f \cdot g' + g \cdot f'$$

$$= \underset{f}{x^2} \cdot \underset{g'}{5x^4} + \underset{g}{x^5} \cdot \underset{f'}{2x} = 5x^6 + 2x^6$$

$$= 7x^6$$

For $f(x) = (8x^5 - 7x + 1)(11 - x^2)$, find $f'(1)$.

first I'll find $f'(x)$, plug in 1.

$$f'(x) = (8x^5 - 7x + 1) \cdot -2x + (11 - x^2) \cdot (40x^4 - 7)$$

so $$f'(1) = (8 - 7 + 1) \cdot -2 + (11 - 1) \cdot (40 - 7)$$

$$= 2 \cdot -2 + 10 \cdot 33$$

$$= -4 + 330 = 326$$

For some function $f(x)$, what is

$$\frac{d^2}{dx^2}\left(x^5 f(x)\right) ?$$

$$\frac{d}{dx}\left(x^5 \cdot f(x)\right) = x^5 \cdot f'(x) + f(x) \cdot 5x^4$$

$$\frac{d^2}{dx^2}\left(x^5 \cdot f(x)\right) = x^5 \cdot f''(x) + f'(x) \cdot 5x^4 + f(x) \cdot 20x^3 + 5x^4 \cdot f'(x)$$

$$\frac{d}{dx}(x^5 + 7x^2) \cdot (8x - x^{10})$$

$$= (x^5 + 7x^2) \cdot (8 - 10x^9) + (8x - x^{10}) \cdot (5x^4 + 14x)$$

Quotients!

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

it's <u>not</u> $\frac{f'}{g'}$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \cdot \left(\frac{f(x+h)\cdot g(x)}{g(x+h)\,g(x)} - \frac{f(x+h)\,g(x+h)}{g(x+h)\,g(x)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{f(x+h)\,g(x) - f(x+h)\,g(x+h)}{g(x+h)\,g(x)}\right)$$

= add & subtract

$$= \lim_{h\to 0} \frac{g(x)\,\frac{f(x+h)-f(x)}{h} - f(x)\,\frac{g(x+h)-g(x)}{h}}{g(x+h)\,g(x)}$$

$$= \boxed{\frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}}$$

$$\boxed{\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{g(x)^2}}$$

The quotient rule.

$$\frac{d}{dx}\frac{hi}{ho} = \frac{ho\ dhi - hi\ dho}{ho\,ho}$$

So

$$\frac{d}{dx}\frac{x^2+2x-1}{3x^5+x} = \frac{(3x^5+x)\cdot(2x+2) - (x^2+2x-1)\cdot(15x^4+1)}{(3x^5+x)^2}$$

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we can do: x^n for n a positive integer,

or any polynomial,

or any product or quotient of a polynomial.

deriv of x^n, n is a negative integer.

$$\frac{d}{dx}x^{-n} = \frac{d}{dx}\left(\frac{1}{x^n}\right)$$

QR

$$= \frac{\cancel{x^n \cdot 0} - 1\cdot n x^{n-1}}{(x^n)^2}$$

$$= \frac{-nx^{n-1}}{x^{2x}} = -n\cdot\frac{x^{n-1}}{x^{2n}}$$

$$= -nx^{n-1-2n} = -nx^{-n-1}$$

so for negative exponents:

$$\frac{d}{dx} x^{-n} = -n x^{-n-1}$$

this is the same rule!

$$\boxed{\frac{d}{dx} x^{n} = n x^{n-1}}$$

actually works for any integer (pos or neg)

How bow non-integers?

actually it works for any real number!

$$\frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-1/2}$$ remember $x^{1/2} = \sqrt{x}$

so $$\frac{d}{dx} \sqrt[3]{x} = \frac{d}{dx}\left(x^{1/3}\right) = \frac{1}{3} x^{-2/3}$$

$$\frac{d}{dx} \sqrt[5]{x^9} = \frac{d}{dx}\left((x^9)^{1/5} \right) = \frac{d}{dx} x^{9/5}$$

$$= \frac{9}{5} x^{4/5}$$

$$\frac{d}{dx} x^{\pi} = \pi x^{\pi - 1}$$

what is that?

$$x^3 = x \cdot x \cdot x$$

$$x^{3.5} = x^1 \cdot x^1 \cdot x^1 \cdot x^{1/2}$$

$$= x^{7/2} = \sqrt{x^7}$$

$$x^{3.61} = x^{\frac{361}{1000}} = \sqrt[1000]{x^{361}}$$

x^{π} means ?

Beware!

$$\frac{d}{dx} x^n = n x^{n-1}$$

but $\frac{d}{dx} 2^x \neq x 2^{x-1}$

2^x is totally different