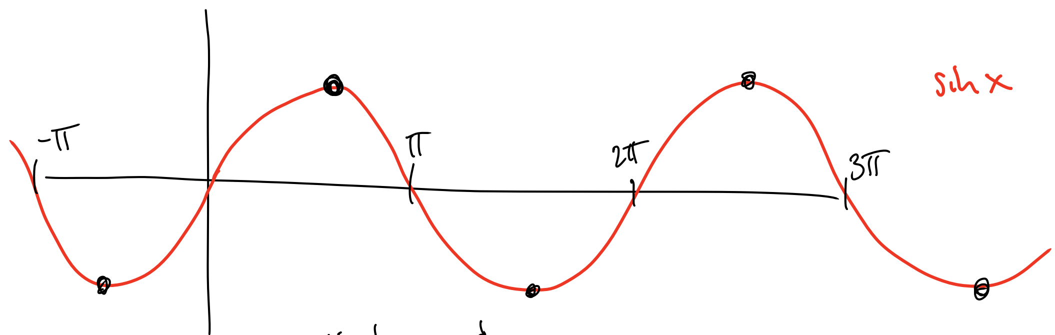
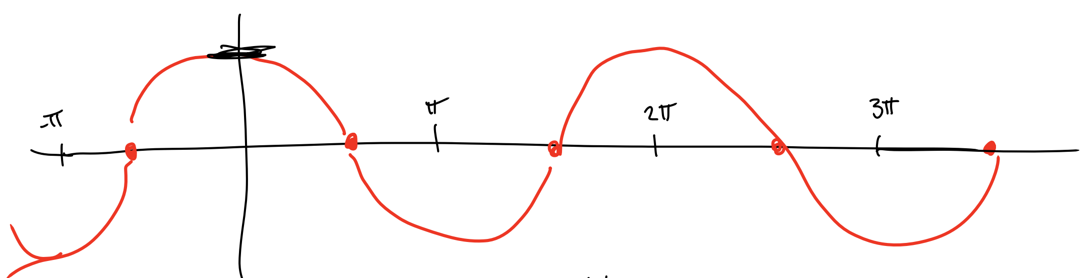
Derivatives of Trigs

deriv. of sin & cos.



let's draw $\frac{d}{dx}\sin x$.



looks like

$$\frac{d}{dx}\sin x = \cos x \; !$$

Let's demonstrate $\frac{d}{dx}\sin x = \cos x$

$$\frac{d}{dx}\sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$\sin(x+h)$ is not $\sin x + \sin h$

$$\boxed{\sin(a+b) = \sin a\cos b + \cos a \sin b}$$

$$\lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

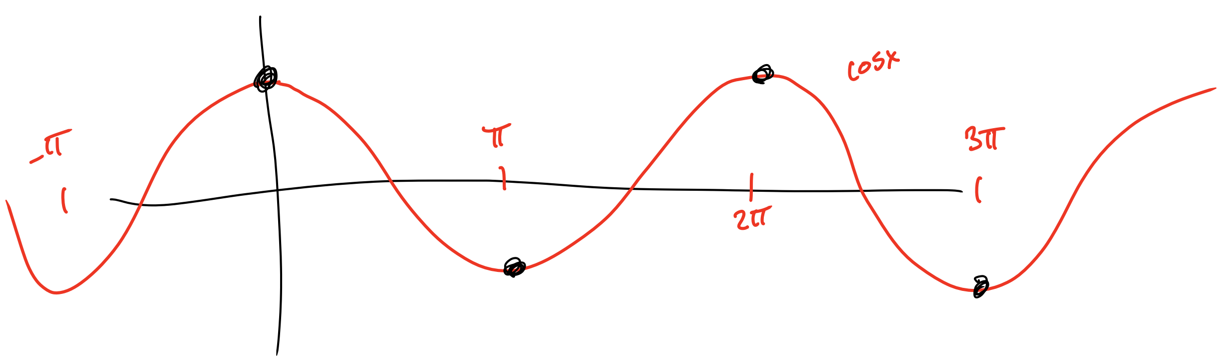
$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

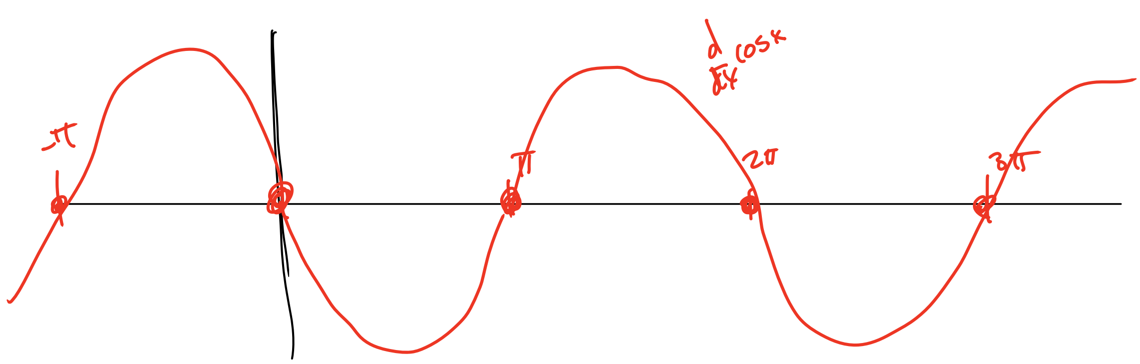
$$= \lim_{h\to 0} \sin x \cdot \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h}$$

$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= \boxed{\cos x}$$

$$\boxed{\frac{d}{dx} \sin x = \cos x}$$





looks like

$$\boxed{\frac{d}{dx}\cos x = -\sin x}$$

$$\frac{d}{dx}\left(x^2 + \sin x + \underbrace{x \cdot \cos x}_{\text{prod rule}} + \frac{\sin x}{x^4}\right)$$

$$= 2x + \cos x + x \cdot -\sin x + \cos x \cdot 1 + \frac{x^4 \cdot \cos x - \sin x \cdot 4x^3}{(x^4)^2}$$

Those other trig functions!

These can all be done as quotients

$$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{(\cos x)^2}$$

$$= \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = \frac{1}{(\cos x)^2}$$

$$= (\sec x)^2 = \sec^2 x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

You derive $\frac{d}{dx}\cot x = -\csc^2 x$

$$\frac{d}{dx}\cot x = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{\sin x \cdot -\sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$= -\csc^2 x$$

$\frac{d}{dx}\sin x = \cos x$	$\frac{d}{dx}\tan x = \sec^2 x$	$\frac{d}{dx}\sec x = \sec x \tan x$
$\frac{d}{dx}\cos x = -\sin x$	$\frac{d}{dx}\cot x = -\csc^2 x$	$\frac{d}{dx}\csc x = -\csc x \cot x$

The Chain Rule

This is about

$$\frac{d}{dx} f(g(x)) = ?$$

like: $\sin(x^2+5x)$

or $(x^7+4x)^{10}$

Idea:

$$\frac{d}{dx} f(g(x)) = \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h\to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h)-g(x)} \cdot \frac{g(x+h)-g(x)}{h}$$

$$= f'(g(x)) \qquad g'(x)$$

$$\boxed{\frac{d}{dx} f(g(x)) = \underbrace{f'(g(x)) \cdot g'(x)}}$$

The Chain Rule

outside

inside

"deriv of outside, with same inside, times deriv. of inside"

$$\frac{d}{dx}\,\underbrace{\sin}_{\text{outside}}\big(\underbrace{x^2+5x}_{\text{inside}}\big) = \cos\left(x^2+5x\right)\cdot\left(2x+5\right)$$

$$\frac{d}{dx}\left(x^7+4x\right)^{10}$$

inside: x^7+4x

outside 10^{th} power

$$= 10\left(x^7+4x\right)^9\cdot\left(7x^6+4\right)$$