Test next Friday!

Chain Rule

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} \cos^3 x = \frac{d}{dx}(\cos x)^3$$

$$= 3(\cos x)^2 \cdot (-\sin x)$$

$$\frac{d}{dx}\sqrt{x\sin} = \frac{d}{dx}\left(\boxed{x}\boxed{\sin x}\right)^{1/2}$$

$$= \frac{1}{2}(x\sin x)^{-1/2} \cdot \left(x \cdot \cos x + \sin x \cdot 1\right)$$

Another form of the chain rule:

Leibniz form:

say y depends on u,

and u depends on x.

Then $\frac{dy}{du}$ is ROC of y with respect to u

$\frac{du}{dx}$ is - - - - u - - - - x

also $\frac{dy}{dx}$ is - - - - y - - - - - - x

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Ex) I'm pumping up a balloon (spherical)

radius obeys:

$$r(t) = 4 - 3/t$$

Give a formula for $\frac{dV}{dt}$ (ROC of volume)

formulas: $r = 4 - 3/t = 4 - 3t^{-1}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = +3t^{-2}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot 3t^{-2}$$

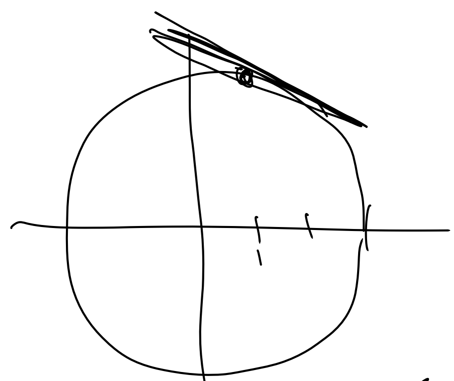
all in terms of t:

$$\frac{dV}{dt} = 4\pi\left(4-3t^{-1}\right)^2 \cdot 3t^{-2}$$

Implicit Differentiation

Finding slopes along weird curves that aren't neccessarily functions.

Slope at top part of circle, radius 3, when $x=1$.



$x^2+y^2=r^2$

$\boxed{x^2+y^2=9}$ not solved for y -

$\boxed{y=x^2+5}$

so y is not explicitly given in terms of x

y is implictly determined by x.

We can take deriv of an implicit formula

Take deriv. of everything on both sides with resp. to x,

The deriv of y becomes $\frac{dy}{dx}$

$$x^2 + y^2 = 9$$

Find $\frac{dy}{dx}$:

deriv: $2x + 2y' \cdot \frac{dy}{dx} = 0$

solve for $\frac{dy}{dx}$:

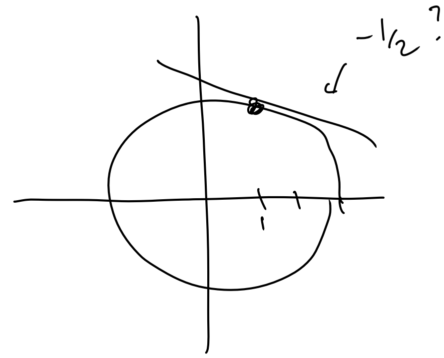
$$2y \cdot \frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

Take deriv of everything - whenever we get to y, multiply by $\frac{dy}{dx}$

Answer will have a mix of x's & y's.

Specifically when $x=1$:



$$\frac{dy}{dx} = \frac{-x}{y}$$

use $x=1$,
for y, do $x^2+y^2=9$

$$1+y^2=9$$
$$y^2=8$$
$$y=\pm\sqrt{8}$$ use $x=1$, $y=\sqrt{8}$

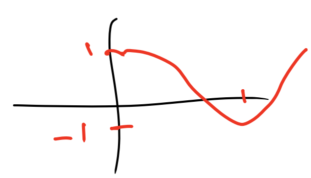
so $\frac{dy}{dx} = \frac{-1}{\sqrt{8}} \approx -.35$

$$\boxed{\sin(xy) = \sin x + \sin y}$$

find slope at $x=\pi$, $y=0$

$$\cos(xy)\cdot\left(x\cdot\frac{dy}{dx} + y\cdot 1\right) = \cos x + \cos y \cdot \frac{dy}{dx}$$

solve for $\frac{dy}{dx}$, then plug in $x=\pi$, $y=0$.



plug first, then solve:

$$\cos(\pi\cdot 0)\left(\pi\cdot\frac{dy}{dx} + 0\cdot 1\right) = \cos\pi + \cos 0\cdot\frac{dy}{dx}$$

$$\cos 0 \left(\pi \frac{dy}{dx} \right) = \cos\pi + \cos 0 \frac{dy}{dx}$$

$$1 \left(\pi \frac{dy}{dx} \right) = -1 + \frac{dy}{dx}$$

$$\pi \frac{dy}{dx} = -1 + \frac{dy}{dx}$$

$$\pi \frac{dy}{dx} - \frac{dy}{dx} = -1$$

$$\frac{dy}{dx}(\pi - 1) = -1$$

$$\frac{dy}{dx} = \frac{-1}{\pi - 1} \approx -1/2$$

Find slope of

$$x^2 - xy - y^2 = 1$$

at $(2,1)$.

$$2x - \left(x \cdot \frac{dy}{dx} + y \cdot 1 \right) - 2y \frac{dy}{dx} = 0$$

plug $x=2$, $y=1$, solve for $\frac{dy}{dx}$

$$2 \cdot 2 - 2 \cdot \frac{dy}{dx} - 1 - 2 \frac{dy}{dx} = 0$$

$$4 - 2y' - 1 - 2y' = 0$$

$$3 = 4y'$$

$y' = 3/4$