Imp. Derivative

The power rule for fractions:

$$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$$

works for any real # n.

Lets demonstrate it for fractions.

Consider $y = x^{p/q}$, p&q are integers

we'll show $\frac{dy}{dx} = \frac{p}{q} x^{p/q - 1}$

$y = x^{p/q}$ do q-th power on both sides:

$y^q = (x^{p/q})^q = x^p$

$y^q = x^p$ do implicit deriv:

$q y^{q-1} \cdot \frac{dy}{dx} = p x^{p-1}$

solve for $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{p x^{p-1}}{q y^{q-1}}$ plug $y = x^{p/q}$

$$= \frac{p\, x^{p-1}}{q\,(x^{p/q})^{q-1}} = \frac{p\, x^{p-1}}{q\, x^{\frac{p}{q}(q-1)}}$$

$$= \frac{p\, x^{p-1}}{q\, x^{p-\frac{p}{q}}} \qquad \frac{x^{7}}{x^{9}}$$

$$= \frac{p}{q}\, x^{p-1-(p-\frac{p}{q})}$$

$$= \frac{p}{q}\, x^{-1+\frac{p}{q}} = \boxed{\frac{p}{q}\, x^{p/q - 1}}$$

so $\frac{dy}{dx} = \frac{p}{q} x^{p/q - 1}$ Shown

$$\frac{5x^2}{(x^2+4x+1)^{1/2}}$$

$$\hookrightarrow \frac{(x^2+4x+1)^{1/2} \cdot 10x - 5x^2 \cdot \frac{1}{2}(x^2+4x+1)^{-1/2} \cdot (2x+4)}{x^2+4x+1}$$