Related Rates

Where one thing depends on another, and both are changing, we want to find how one rate of change depends on the other.

Ex1 I dump paint on the floor, it spreads out in a circle, area increases at 50 cm^2/sec.

How fast is the radius increasing when radius is 10 cm?

Quantities are area & radius.

formula:

$$A = \pi r^2$$

take implicit deriv $\frac{d}{dt}$

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

We'll solve for: $\frac{dr}{dt}$

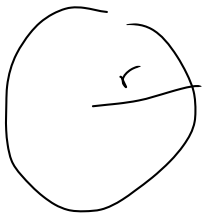
plug $\frac{dA}{dt} = 50$

$r = 10$

$50 = 2\pi \cdot 10 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{50}{2\pi \cdot 10} = .79$ cm/sec

At that same moment, how fast is the perimeter increasing?



Formula relating radius & perimeter

$P = 2\pi r$

deriv: $\frac{dP}{dt} = 2\pi \frac{dr}{dt}$

Solve for $\frac{dP}{dt}$ plug in $\frac{dr}{dt} = .79$

so $\frac{dP}{dt} = 2\pi \cdot .79 = 5$ cm/sec

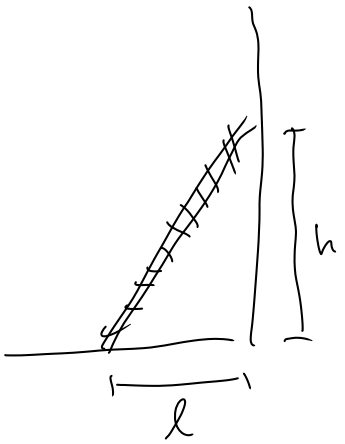
General Strategy

- Figure out which quantities are changing, & give them variable names. A & r
- relate the variables using an equation $A = \pi r^2$ (don't plug in yet!)
- Take implicit deriv.
- Decide what to solve for, plug in everything else.

Sliding ladder!

A 10-foot ladder is sliding down the wall.

Bottom legs move away from the wall at 1 ft/sec.



How fast is the top moving down when the bottom is 3 ft away from the wall?

Equation:

$$\ell^2 + h^2 = 10^2$$

Imp deriv:

$$2\ell \frac{d\ell}{dt} + 2h \frac{dh}{dt} = 0$$

solve for $\frac{dh}{dt}$, plug in: $\frac{d\ell}{dt} = 1$

$\ell = 3$

$h = \sqrt{91}$

finds the h using $\ell^2 + h^2 = 10^2$ with $\ell = 3$.

$3^2 + h^2 = 10^2$

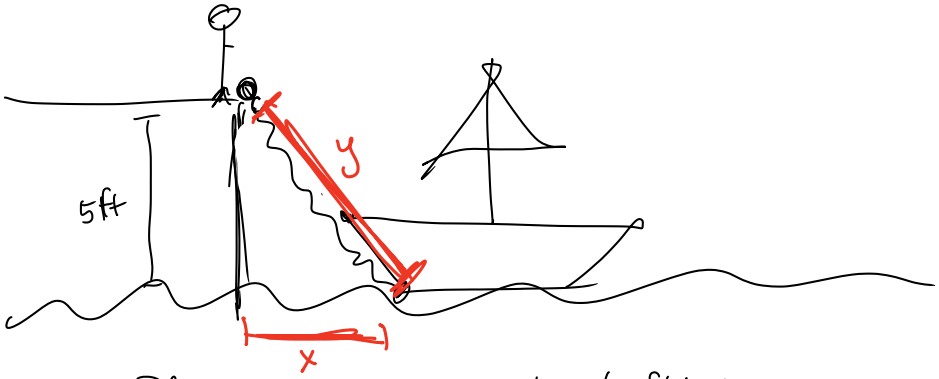
$9 + h^2 = 100$

$h^2 = 91$ $\quad h = \sqrt{91}$

$$2 \cdot 3 \cdot 1 + 2\sqrt{91} \frac{dh}{dt} = 0$$

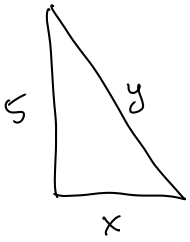
$$6 + 2\sqrt{91} \frac{dh}{dt} = 0$$

$$\frac{dh}{dt} = \frac{-6}{2\sqrt{91}} = -.31 \text{ ft/sec}$$



If I pull the rope at (½ ft/sec)
how fast is the boat moving towards the dock when it's 10 ft away?

Changing quantities: distance from boat to dock



Equation:

$$x^2 + 5^2 = y^2$$

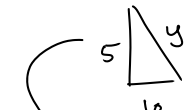
Deriv: $2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

Solve for: $\frac{dx}{dt}$

plug: $x = 10$

$y = \sqrt{125}$

$\frac{dy}{dt} = -1/2$



$5^2 + 10^2 = y^2$

$125 = y^2$

$y = \sqrt{125}$

$$2 \cdot 10 \cdot \frac{dx}{dt} + 0 = 2 \cdot \sqrt{125} \cdot \frac{1}{2}$$

$$20 \frac{dx}{dt} = -\sqrt{125}$$

$$\frac{dx}{dt} = \frac{-\sqrt{125}}{20} = -.55 \text{ ft/sec}$$

moves at .55 ft/sec towards the dock.