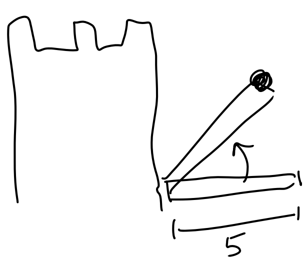
Related Rates

- Identify 2 changing quanties, give them names
- Write a formula relating them
- Do impl. deriv
- Choose what to solve for, plug in everything else.

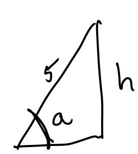
Ex1 The drawbridge is 5 ft long.



The top edge is rising by 6 in/sec.

How fast is the angle changing when it's $45°$?

2 Quanties: height of the top, h
angle. a



Formula relating a & h:

$$\sin a = h/5$$

Imp deriv: $\frac{d}{dt}$ $\qquad$ $\frac{1}{5}h$

$$\cos a \cdot \frac{da}{dt} = \frac{1}{5}\frac{dh}{dt}$$

Solve for $\frac{da}{dt}$, $\qquad$ plug $\quad a = \pi/4$

$\frac{dh}{dt} = 6$

$$\frac{d}{dx}\sin x = \cos x$$

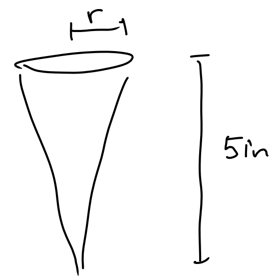
$$\cos \pi/4 \cdot \frac{da}{dt} = \frac{1}{5} \cdot 6$$

$$\frac{1}{\sqrt{2}} \frac{da}{dt} = \frac{6}{5}$$

$$\frac{da}{dt} = \frac{6}{5}\sqrt{2} = .141 \ \frac{\text{radians}}{\text{sec}}$$

$\rightarrow$ 8.1 degrees/sec

Dripping Icicle:



As it drips, the length stays same, but the radius shrinks

It drips at $1 \ \text{in}^3/\text{hr}$ $\qquad$ $\left(V = \frac{1}{3}\pi r^2 h\right)$

How fast is the radius changing when it is $\frac{1}{2}$ in?

$$V = \frac{1}{3}\pi r^2 \cdot 5$$

$$V = \frac{5}{3}\pi r^2$$

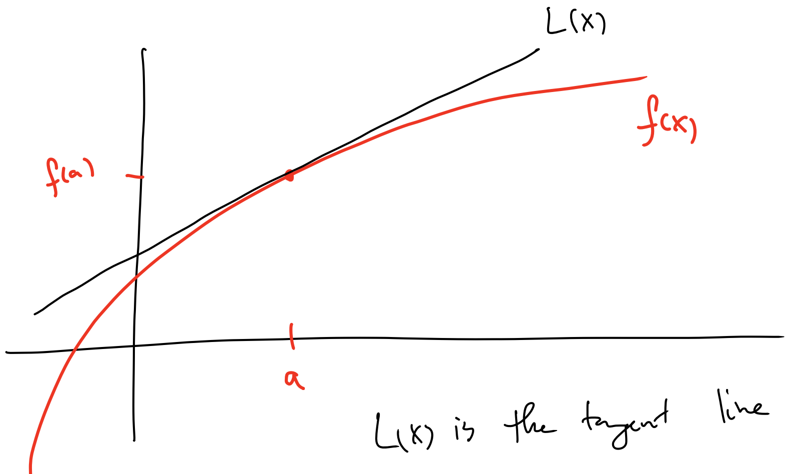
$$\underset{-1}{\frac{dV}{dt}} = \frac{5}{3}\pi \cdot 2\underset{1/2}{r}\ \underset{\text{solve}}{\frac{dr}{dt}}$$

$$-1 = \frac{5}{3}\pi \cdot \cancel{2 \cdot \frac{1}{2}} \cdot \frac{dr}{dt}$$

$$-\frac{3}{5\pi} = \frac{dr}{dt}$$

$$\boxed{\frac{dr}{dt} = -.19 \ \text{in/hr}}$$

Linear Approximation



L(x) is the tangent line to f(x) at x=a.

Equation of L(x): point-slope form:

$$y - y_0 = m(x - x_0)$$

$$y - f(a) = f'(a)(x-a)$$

$$y = f(a) + f'(a)(x-a)$$

So $\boxed{L(x) = f(a) + f'(a)(x-a)}$

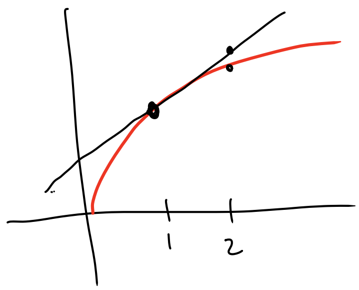
the linear approximation of f(x) at x=a.

It's useful because

$L(x) \approx f(x)$ when x is near a.

Let's estimate $\sqrt{2}$ using lin. approx.

let $f(x) = \sqrt{x}$, choose $a = 1$.



$L(x) = f(a) + f'(a)(x-a)$

$= f(1) + f'(1)(x-1)$

$f(1) = \sqrt{1} = 1$

$f'(1)$: $f'(x) = \frac{1}{2}x^{-1/2}$

so $f'(1) = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}$.

so $L(x) = 1 + \frac{1}{2}(x-1)$

so $\sqrt{2} = f(2) \approx L(2) = 1 + \frac{1}{2}(2-1)$

$= 1 + \frac{1}{2}\cdot 1 = 1.5$

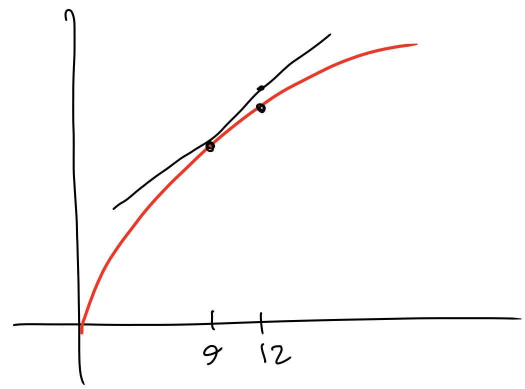
$\sqrt{2} = 1.414\ldots$

Choose the a to be some nearby value where you can compute $f(a)$

Let's approximate $\sqrt{12}$

Use $f(x) = \sqrt{x}$

Choose $a = 9$



$$L(x) = f(a) + f'(a)(x-a)$$
$$= f(9) + f'(9)(x-9)$$

$$f(9) = \sqrt{9} = 3$$
$$f'(9) = \frac{1}{2}(9)^{-1/2} = \frac{1}{2} \cdot \frac{1}{9^{1/2}} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

So $L(x) = 3 + \frac{1}{6}(x-9)$

$$\sqrt{12} = f(12) \approx L(12)$$
$$= 3 + \frac{1}{6}(12-9)$$
$$= 3 + \frac{1}{6} \cdot 3 = 3.5$$

$\sqrt{12} = 3.46\ldots$