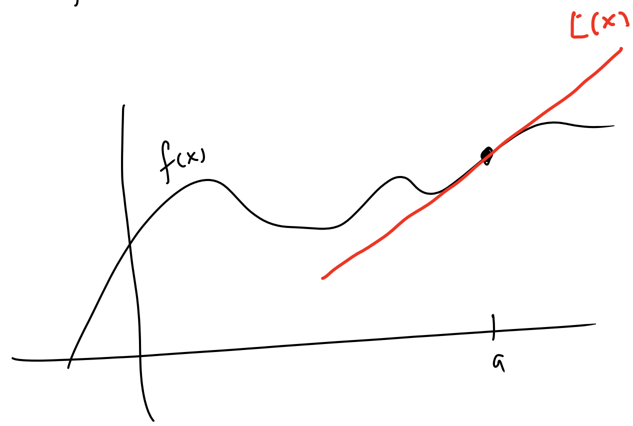
Linear Approximation

$$L(x) = f(a) + f'(a)(x-a)$$

"The lin. approx of $f(x)$ near $x=a$"



useful to approximate $f(x)$,

$L(x) \approx f(x)$ when x is near a.

Let's estimate $\sqrt{27}$

we'll plug in $L(27)$

$$L(x) = f(a) + f'(a)(x-a)$$

choose $f(x) = \sqrt{x}$ $\quad a = 25$

$= x^{1/2}$

some point near 27 which we already know square root of.

$f(a) = \sqrt{25} = 5$

for $f'(a)$, find $f'(x)$, then plug in 25.

$f'(x) = \frac{1}{2} x^{-1/2}$ $\quad f'(25) = \frac{1}{2} \cdot 25^{-1/2}$

$= \frac{1}{2} \cdot \frac{1}{25^{1/2}} = \frac{1}{2} \cdot \frac{1}{5}$

$= 1/10$

$L(x) = f(a) + f'(a)(x-a)$

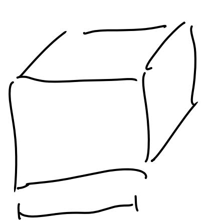
$= 5 + \frac{1}{10}(x-25)$

So $L(27) = 5 + \frac{1}{10}(27-25)$

$= 5 + \frac{1}{10} \cdot 2 = 5.2$

actually, $\sqrt{27} = 5.1961\ldots$

Use a linear approx to estimate the suface area of a cube with volume 9



x

$\sqrt[3]{9}$

if volume $= 9$,

the side length is: $\sqrt[3]{9}$

So the surface area is: $6 \cdot \underbrace{(\sqrt[3]{9})^2}_{\text{area of each side}}$

We need to estimate

$$6\left(\sqrt[3]{9}\right)^2 = \boxed{6 \cdot 9^{2/3}}$$

$f(x) = 6x^{2/3}$

$f'(x) = 4x^{-1/3}$

$a = 8$

something near 9 which we know the $2/3$ - power of.

$$L(x) = f(a) + f'(a)(x-a)$$

$$f(a) = 6 \cdot 8^{2/3} = 6 \cdot \left(\sqrt[3]{8}\right)^2 = 6 \cdot 2^2 = 6 \cdot 4 = 24$$

$$f'(a) = 4 \cdot 8^{-1/3} = 4 \cdot \frac{1}{8^{1/3}} = 4 \cdot \frac{1}{2} = 2$$

$$L(x) = 24 + 2(x-8)$$

so $6 \cdot 9^{2/3} \approx L(9) = 24 + 2(9-8)$

$= 26$

actual: $6 \cdot 9^{2/3} = 25.96\ldots$

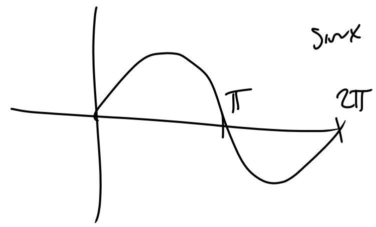
You try: Estimate $\sin 3$

3 radians.

with linear approx.

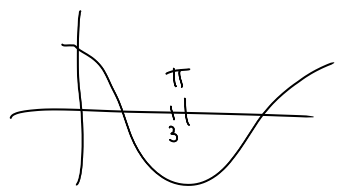
$f(x) = \sin x$ $\qquad a = \pi$

$f'(x) = \cos x$



$L(x) = f(a) + f'(a)(x-a)$

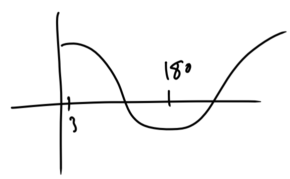
$= \sin\pi + \cos\pi\,(x-\pi)$



$= 0 + -1(x-\pi)$

$\boxed{L(x) = \pi - x}$

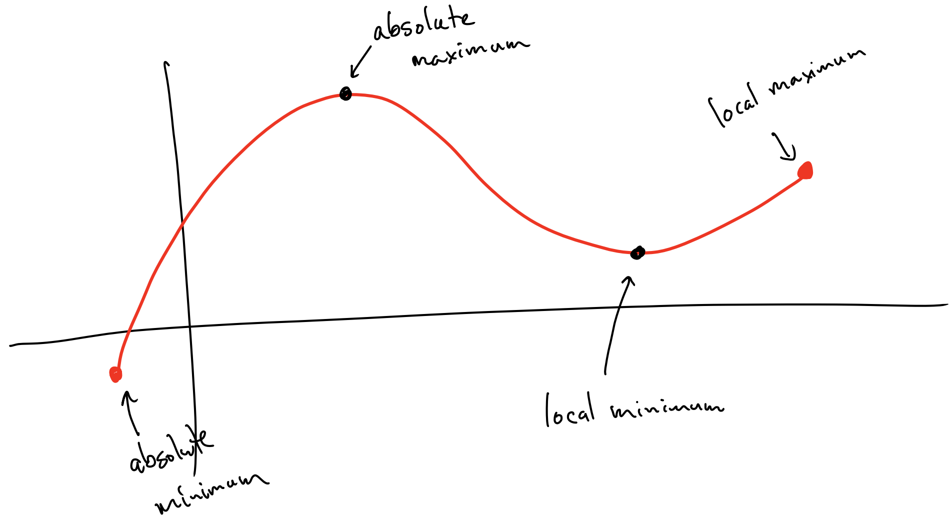
So $L(3) = \pi - 3 = .1415\ldots$



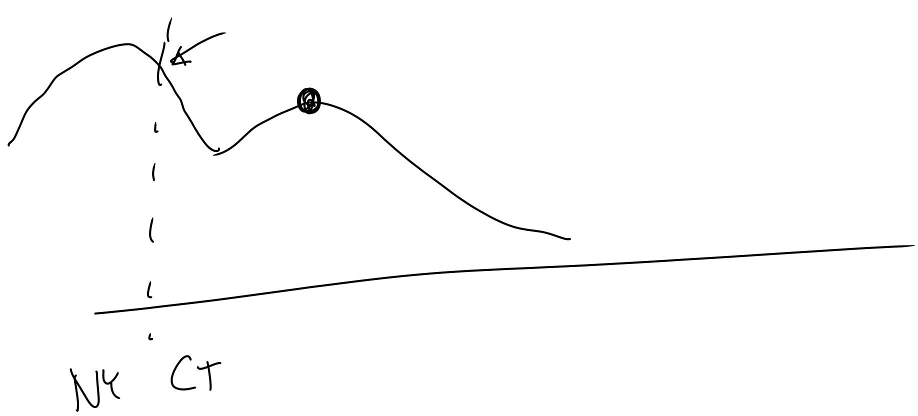
actually $\sin 3 \approx .1411\ldots$

$\sin 3^\circ$? this time use $a = 0$.

Maximum & Minimum Values



abs max means some point where the y-value is greater or equal to all other y-values.



Def For a function with domain D,

$f(c)$ is an <u>absolute</u> <u>maximum</u> if $f(c) \geq f(x)$ for every x in D.

- - - - - - <u>minimum</u> ... $f(c) \leq f(x)$ - - - - -

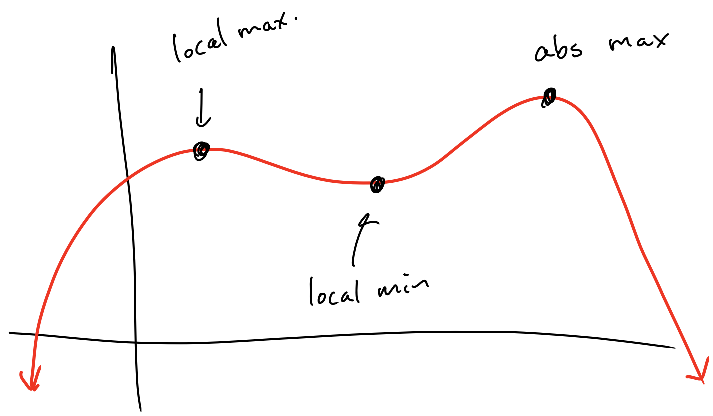
$f(c)$ is a <u>local</u> <u>maximum</u> if $f(c) \geq f(x)$ for all x near c.

- - - - - <u>minimum</u> - - $f(c) \leq f(x)$ - - - - - .

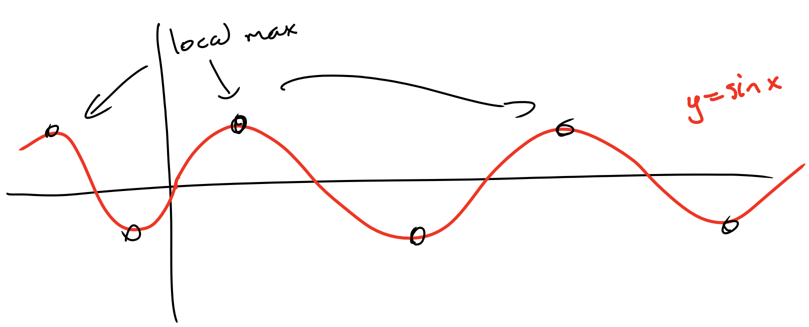
every absolute max/min is automatically a local max/min.

Any max/min value is called an <u>extreme</u> <u>value</u>

or extremum.

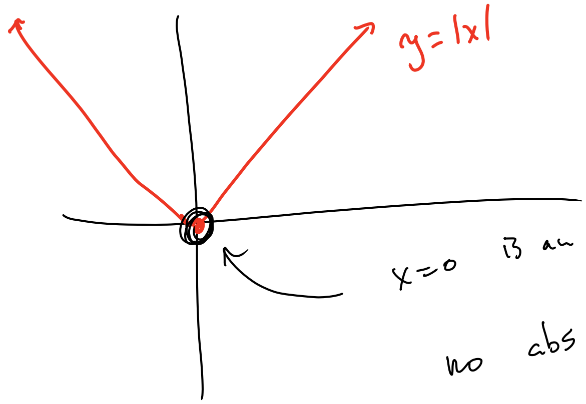


no absolute min.



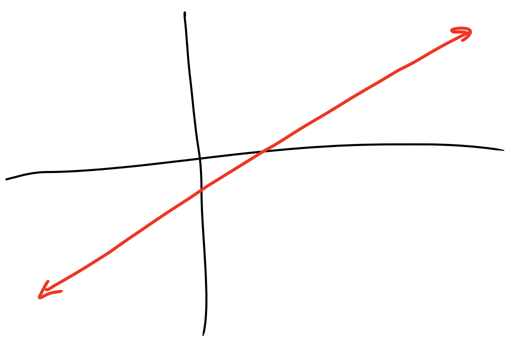
a local max at $\pi/2 + 2\pi n$, these are all absolute max.

local min at $3\pi/2 + 2\pi n$, these are all absolute min.



$x = 0$ is an abs min,

no abs max.

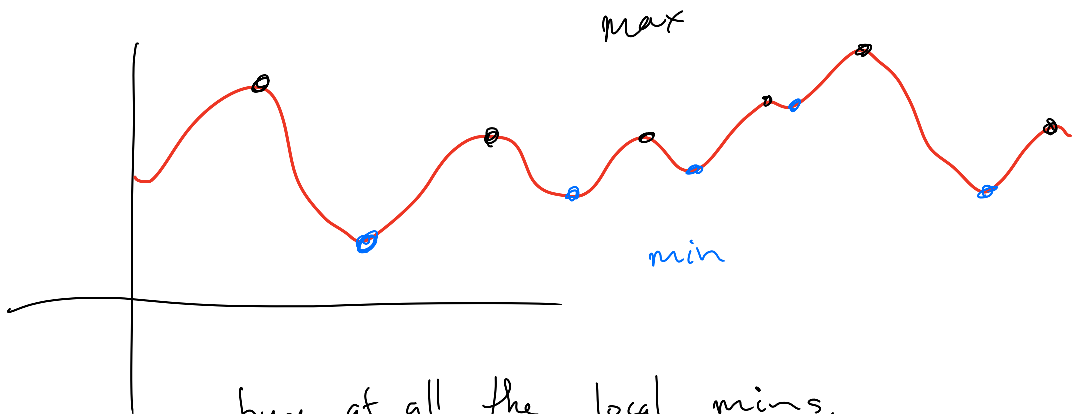


no extreme values.

Finding extreme values is super-important

"I throw a rock - how high does it go?"

"what pricing will maximize my profit?"



buy at all the local mins,
sell at all the local maxs.

We'll learn methods to find
abs & local extrema.