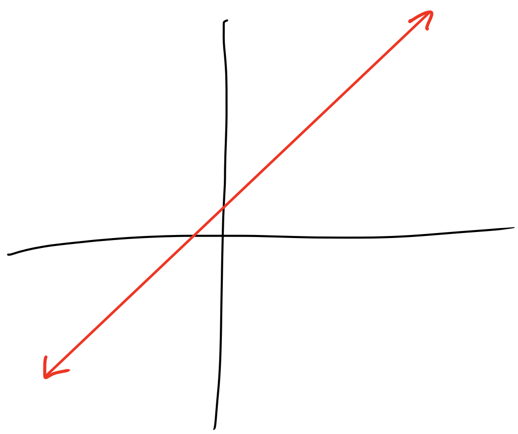
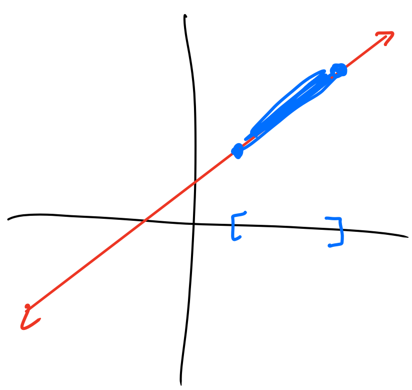
Absolute extrema

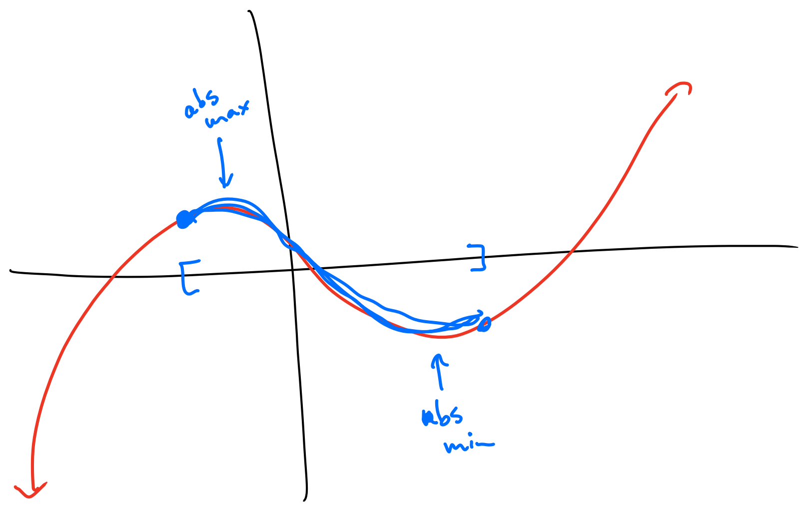
easier to identify than locals,
but often they don't exist



← no abs max or min.

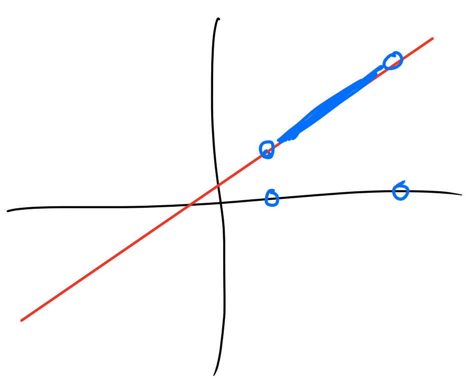
<u>But</u> if we focus only on a closed interval in the domain, then it does have absolute extrema



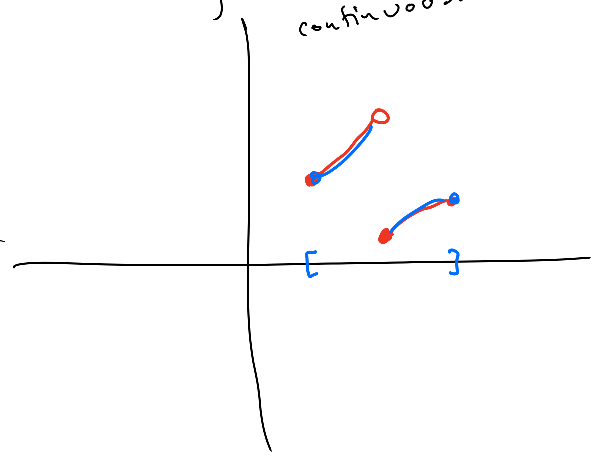


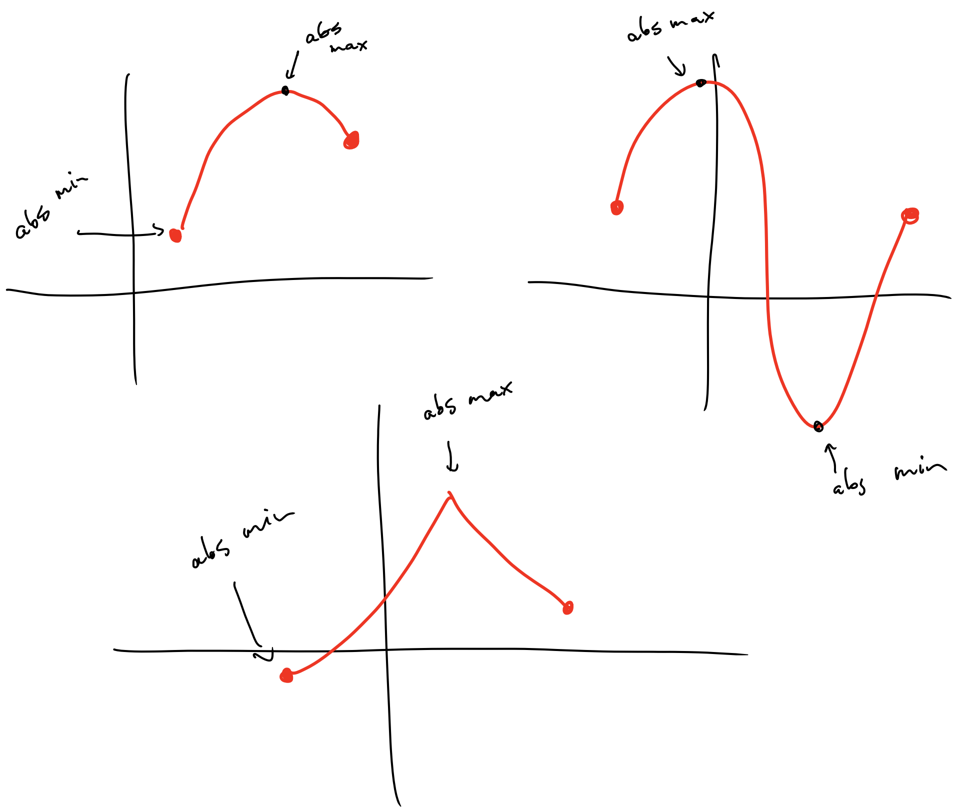
The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f has an abs max & an abs min on $[a, b]$

Interval must be closed:



f must be continuous.





Notice: abs extrema occur:

- At the endpoint

or • at some point where $f'(x) = 0$

or • at some poin where $f'(x)$ DNE.

$\Big\}$ ← critical numbers

Def A critical number of f is some point c in the domain where $f'(c) = 0$ or $f'(c)$ DNE.

To find abs. extrema:

The Closed Interval Method

When f is continuous on $[a,b]$,

we check all values of $f(x)$ at

any critical #s & at the endpoints.

The biggest & smallest of these are

the absolute min & max.

Ex1 Find abs extrema for

$$f(x) = 2x^3 - 18x^2 - 6 \quad \text{on} \quad [-1, 10]$$

Find critical #s:

$$f'(x) = 6x^2 - 36x$$

$f' = 0$: $\quad 6x^2 - 36x = 0$

$$6x(x-6) = 0$$

$$\boxed{x = 0, \quad x = 6}$$

f' DNE N/A $\quad$ f' always exists

$x=0$, $x=6$ are the only critical #s.

plug in $x=0$, $x=6$, $x=-1$, $x=10$ to $f(x)$

$$f(x) = 2x^3 - 18x^2 - 6$$

calc:

crit #s

$f(0) = -6$

$f(6) = -222 \leftarrow$ min

endpoints

$f(-1) = -26$

$f(10) = 194 \leftarrow$ max

The abs max is $f(10) = 194$

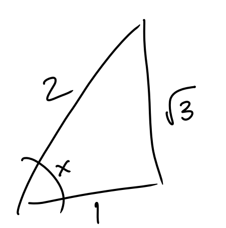
abs min is $f(6) = -222$

Find abs extrema of

$$f(x) = x - 2\sin x \quad \text{for} \quad 0 \leq x \leq 2\pi$$

critical #s:

$f'(x) = 1 - 2\cos x$

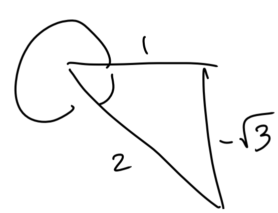


$\underline{f'=0}$ $\quad 1 - 2\cos x = 0$

$1 = 2\cos x$

$\cos x = 1/2$

$x = \pi/3$ or $\dfrac{5\pi}{3}$



all other answers for x
are outside the interval $[0, 2\pi]$

f' DNE N/A

plug in: $f(x) = x - 2\sin x$

min

$f(\pi/3) = \pi/3 - 2\sin \pi/3 = \pi/3 - 2 \cdot \frac{\sqrt{3}}{2} = \pi/3 - \sqrt{3} = -.684\ldots$

$f(5\pi/3) = 5\pi/3 - 2\sin 5\pi/3 = 5\pi/3 - 2 \cdot -\frac{\sqrt{3}}{2} = 5\pi/3 + \sqrt{3} = 6.968\ldots$

max

$f(0) = 0 - 2\sin 0 = 0$

$f(2\pi) = 2\pi - 2\sin 2\pi = 2\pi - 0 = 2\pi = 6.28\ldots$

abs max is $f(5\pi/3) = 6.968\ldots$

abs min is $f(\pi/3) = -.684\ldots$

Find abs extrema of

$$f(x) = (x^2 - 4)^3 \quad \text{on} \quad [-2, 3]$$

$$f'(x) = 3(x^2-4)^2 \cdot 2x$$

$$= 6x(x^2-4)^2$$

$f' = 0$: $\quad 6x(x^2-4)^2 = 0$

$6x = 0 \quad$ or $\quad (x^2-4)^2 = 0$

$x=0$ $\qquad$ $x^2-4=0$ $\qquad$ $(x^2-4)^3$

$(x+2)(x-2)=0$

$x=-2$, $x=2$

$f(0) = (0-4)^3 = -64 \leftarrow$

$f(-2) = ((-2)^2-4)^3 = 0$

$f(2) = (2^2-4)^3 = 0$

$f(3) = (3^2-4)^3 = 5^3 = 125 \leftarrow$

abs min at $f(0) = -64$

abs max at $f(3) = 125.$