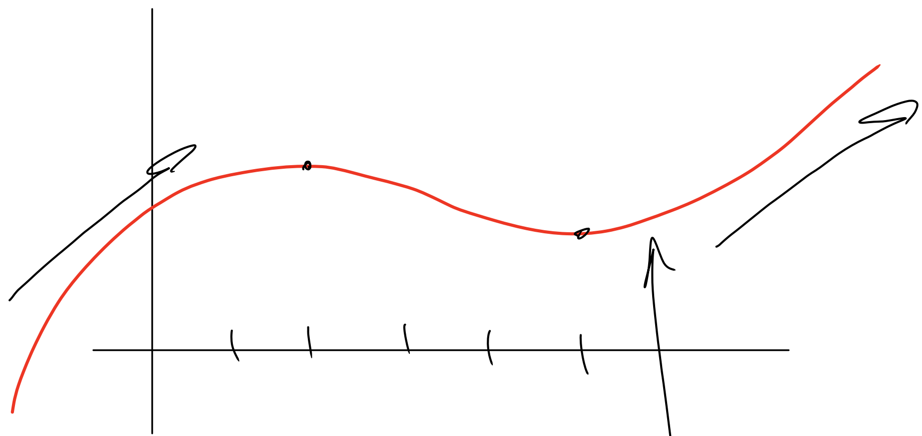


f is inc. when $f'(x) > 0$

dec when $f'(x) < 0$

inc for x in $(-\infty, 2)$

and $(5, \infty)$

decreasing for x in $(2, 5)$

Just algebraically:

First find the critical #s,

plug in values in between to see

where f' is pos vs neg.

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Say where it's inc & dec.

$f'(x) = 12x^3 - 12x^2 - 24x$ simplify

$= 12x(x^2 - x - 2 \quad)$

$$\boxed{f'(x) = 12x(x-2)(x+1)}$$

<u>crit #s</u>

<u>$f'=0$</u> $\qquad 12x(x-2)(x+1) = 0$

$12x = 0 \qquad x-2=0 \qquad x+1=0$

$x=0 \qquad x=2 \qquad x=-1 \quad \leftarrow$

<u>f' DNE</u> (N/A)

Crit #s: $0, -1, 2.$ $\qquad \boxed{f'(x) = 12x(x-2)(x+1)}$

x:		-1		0		2	
f':	−	0	+	0	−	0	+

is it pos/neg in between?

Choose "test points" in each little interval.

$f'(1) = 12\cdot 1(1-2)(1+1)$
$+ \cdot + \cdot - \cdot + = -$

$f'(3) = 12\cdot 3(3-2)(3+1)$
$+ \ + \ + \ +$

$f'(-2) = 12\cdot(-2)(-2-2)(-2+1)$
$+ \ - \ - \ -$

$f'(-1/2) = 12(-1/2)(-1/2-2)(-1/2+1)$

$+ \quad - \quad - \quad +$

So f is inc on $(-1, 0)$ & $(2, \infty)$

f is dec on $(-\infty, -1)$ & $(0, 2)$

Ex1 Where is it inc/dec?

$$f(x) = \frac{x^2-7}{x-4}$$

$$f'(x) = \frac{(x-4)\cdot 2x - (x^2-7)\cdot 1}{(x-4)^2}$$

$$= \frac{2x^2-8x-x^2+7}{(x-4)^2}$$

$$= \frac{x^2-8x-7}{(x-4)^2} = \boxed{\frac{(x-7)(x-1)}{(x-4)^2}}$$

$f'=0$ $\quad \frac{(x-7)(x-1)}{(x-4)^2} = 0$

$$(x-7)(x-1) = 0 \cdot (x-4)^2$$

$$(x-7)(x-1) = 0$$

$x=7$, $x=1$

f' DNE

denom = 0

$(x-4)^2 = 0$

$x - 4 = 0$

$x = 4$

$$= \boxed{\frac{(x-7)(x-1)}{(x-4)^2}}$$

	1		4		7	
f': +	0	−	DNE	−	0	+

$$f'(0) = \frac{(0-7)(0-1)}{(0-4)^2} = \frac{- \cdot -}{+} = +$$

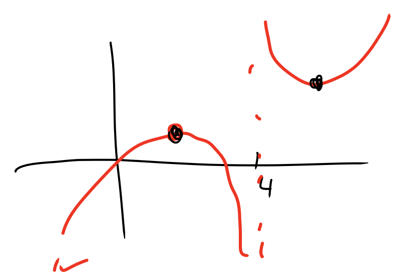
$$f'(2) = \frac{(2-7)(2-1)}{(\ \ )^2} = \frac{- \cdot +}{+} = -$$

$$f'(5) = \frac{(5-7)(5-1)}{(\ \ )^2} = \frac{- \cdot +}{+} = -$$

$$f'(8) = \frac{(8-7)(8-1)}{(\ \ )^2} = \frac{+ \cdot +}{+} = +$$

inc on $(-\infty, 1)$ & $(7, \infty)$

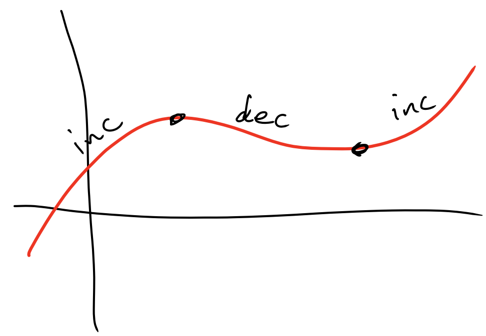
dec on $(1, 4)$ & $(4, 7)$



We can use that same chart to identify local min & maxs.

Thm <u>The First Derivative Test</u>

Let c be a critical number of $f(x)$, and assume f is continuous at c. Then:



- If f goes from inc to dec near c, then c is a local max.
- " " " " dec to inc " " " " " " " " " min.
- If f goes inc to inc or dec to dec, then c is not a local max or min.

Previous example:

		1		4		7	
f':	$+$	0	$-$	DNE	$-$	0	$+$

So $x=1$ is a local max (goes $+$ to $-$)

$x=7$ is a local min ($-$ to $+$)

($x=4$ is not a min or max)

Ex

$f(x) = 3x^4 + 4x^3 + 1$

find intervals where it's inc/dec,

find x-values for local extrema.

$f'(x) = 12x^3 + 12x^2$

$= \boxed{12x^2(x+1)}$

$f' = 0$ $\quad x = 0, \quad x = -1$

f' DNE $\quad$ n/a

x		-1		0	
$f'(x)$	$-$	0	$+$	0	$+$

$f'(-2) = 12 \cdot (-2)^2(-2+1)$
$\quad + \quad + \quad -$

$f'(-1/2) = 12 \cdot (-1/2)^2(-1/2+1)$
$\quad + \quad + \quad +$

$f'(1) = 12 \cdot 1^2(1+1)$

inc on $(-1, 0)$ & $(0, \infty)$

dec on $(-\infty, -1)$

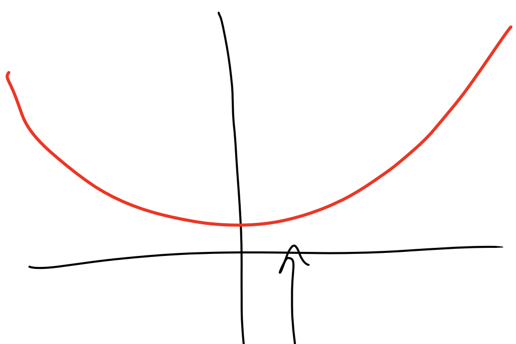
$x = -1$ is a local min.

f'' is also visible graphically

[f' is pos means f is increasing]

f'' is pos means f' is increasing

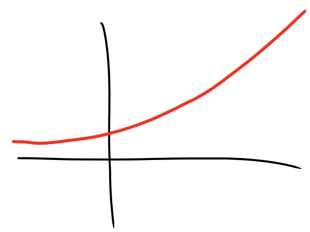
slope is becoming bigger & bigger as we move right.

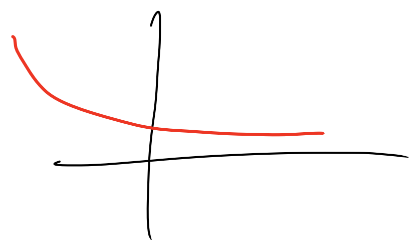


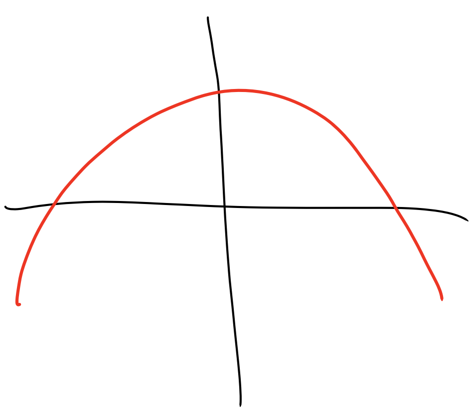
this has $f'' > 0$

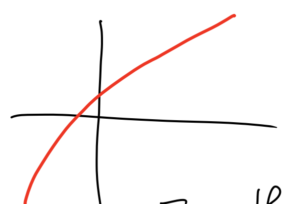
"concave up"

or



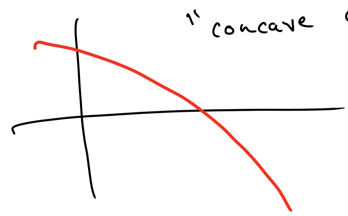






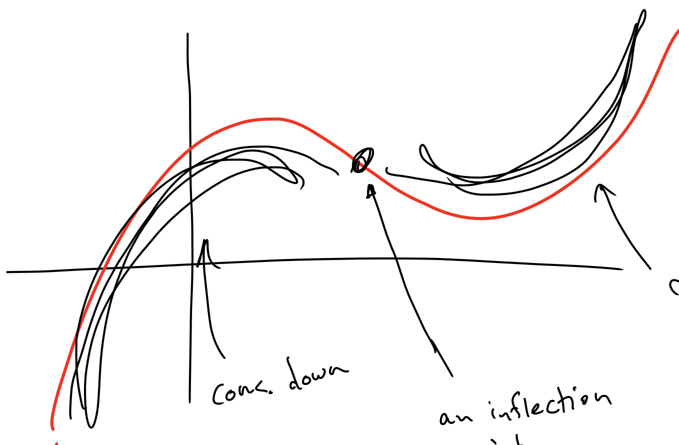
For these, $f'' < 0$

"concave down"

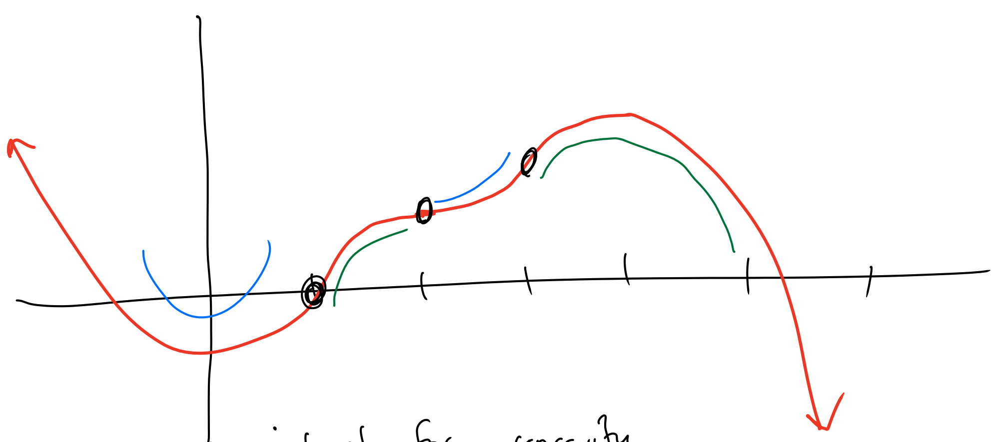


Concave up:  f'' is pos

Concave down:  f'' is neg



($f''=0$)



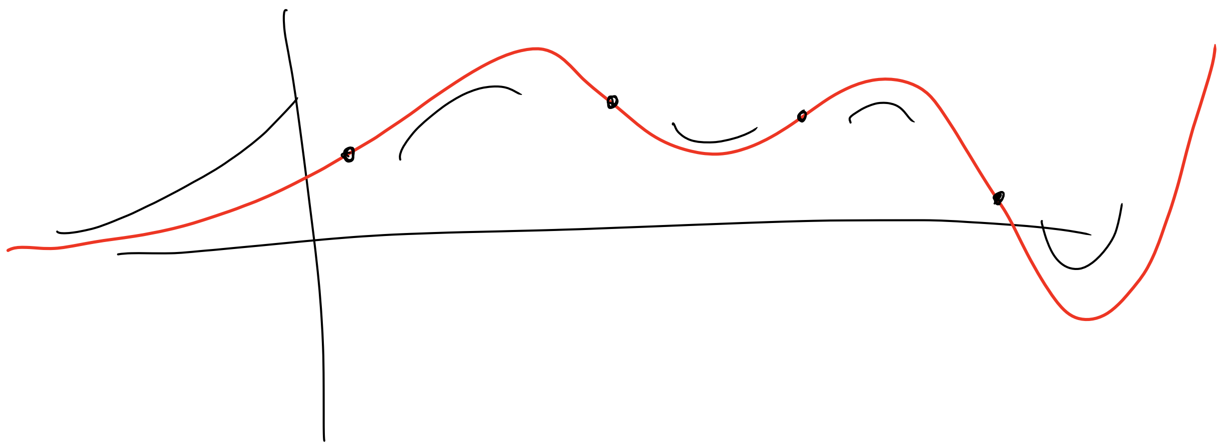
intervals for concavity

concave up: $(-\infty, 1)$ & $(2, 3)$

concave down: $(1, 2)$ & $(3, \infty)$

inflection pts at $x = 1, 2, 3$

Note: inc: $(0,2)$ & $(2,4)$
dec: $(-\infty,0)$ & $(4,\infty)$



intervals of concavity using the formula

do same procedure as inc/dec,
but use f'' instead of f'.

Ex1 $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
find intervals where it's concave up/down.

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$f''(x) = 36x^2 - 24x - 24$$
$$= \boxed{12(3x^2 - 2x - 2)}$$

$f'' = 0$: $\quad 12(3x^2 - 2x - 2) = 0$

$$3x^2 - 2x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4 \cdot 3 \cdot -2}}{2 \cdot 3}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{6} = \frac{2 \pm \sqrt{28}}{6}$$

$$= -.548,\ 1.215$$

$$\boxed{12(3x^2 - 2x - 2)}$$

$-.548$ $\quad$ 1.215

f'': $\quad + \quad 0 \quad - \quad 0 \quad +$

$$f''(-1) = 12(3 \cdot (-1)^2 - 2(-1) - 2)$$
$$= 12(3 + 2 - 2)$$
$$= 12 \cdot 3$$

$$f''(0) = 12(3 \cdot 0^2 - 2 \cdot 0 - 2)$$
$$12 \cdot -2$$

$$f''(2) = 12(3 \cdot 2^2 - 2 \cdot 2 - 2)$$
$$= 12(12 - 4 - 2)$$
$$+ \quad\quad +$$

concave up: $(-\infty, -.548)$ & $(1.215, \infty)$

concave down: $(-.548, 1.215)$