

Curve Sketching

using the inc/dec intervals & conc up/down,
we can draw the graph!

Ex/

$$f(x) = x^4 - 4x^3 \quad \text{draw the graph.}$$

find intervals of inc/dec & conc. up/down

inc/dec

critical #s: $f'(x) = 4x^3 - 12x^2$
 $= 4x^2(x-3)$

$$f' = 0: \quad 4x^2(x-3) = 0$$

$$x=0 \quad x=3$$

$$f': \quad \begin{array}{c} 0 \qquad 3 \\ \hline - \quad 0 \quad - \quad 0 \quad + \end{array}$$

$$f'(-1) = 4(-1)^2(-1-3)$$

+ + -

$$f'(1) = 4(1)^2(1-3)$$

+ + -

$$f'(4) = 4(4)^2(4-3)$$

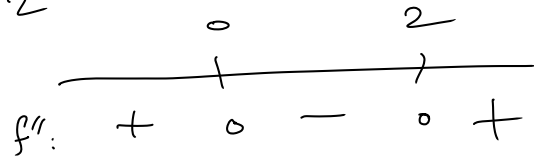
+ + +

Concave up/down: $f'(x) = 4x^3 - 12x^2$

$$f''(x) = 12x^2 - 24x$$

$$= 12x(x-2)$$

$$f'' = 0: \quad x=0, \quad x=2$$



$$f''(-1) = 12(-1)(-1-2)$$

+ - -

$$f''(1) = 12(1)(1-2)$$

+ + -

$$f''(3) = 12(3)(3-2)$$

+ + +

Plot the critical #s & inflection pts
with their y-values.

$$\text{crit: } \quad x=0, \quad x=3$$

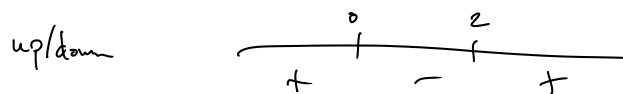
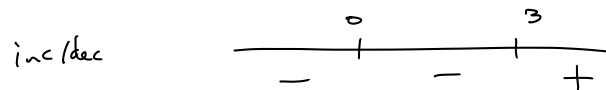
$$\text{infl: } \quad x=0, \quad x=2$$

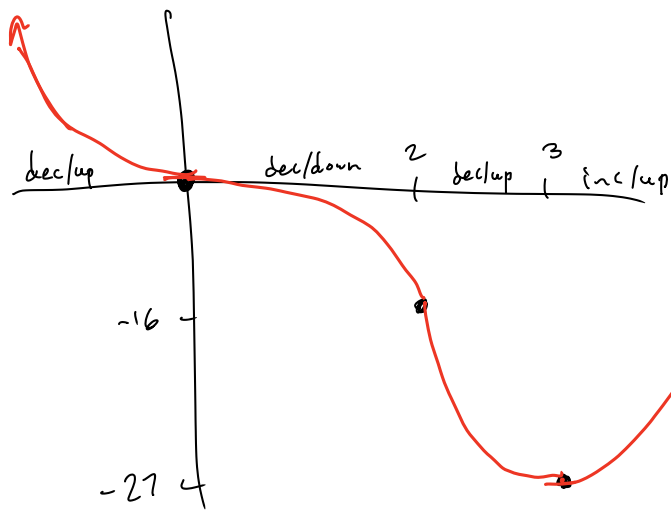
$$\text{y-values: plug in to } f(x) = x^4 - 4x^3$$

$$f(0) = 0^4 - 4 \cdot 0^3 = 0$$

$$f(3) = 3^4 - 4 \cdot 3^3 = -27$$

$$f(2) = 2^4 - 4 \cdot 2^3 = -16$$





draw in the curves
using inc/dec & up/down

Shapes go like

	inc	dec
up	↗	↘
down	↖	↙

$$f(x) = x^3 - 6x^2 - 135x$$

inc/dec $f'(x) = 3x^2 - 12x - 135$

$$= 3(x^2 - 4x - 45)$$

$$= \boxed{3(x - 9)(x + 5)}$$

crit #s: $x=9, x=-5$

	-5		9	

f':	+	0	-	0
		+		

$$f'(-6) = 3(-6-9)(-6+5)$$

+ - -

$$f'(0) = 3(0-9)(0+5)$$

+ - +

$$f'(10) = 3(10-9)(10+5)$$

+ + +

conc. up/down $f'(x) = 3x^2 - 12x - 135$

$$f''(x) = 6x - 12$$

$$= 6(x-2)$$

$$f'' = 0: \quad x = 2$$

	2	
----- -----		
-	0	+

f''

$$f''(0) = 6 \cdot 0 - 12 = -$$

$$f''(3) = 6 \cdot 3 - 12 = +$$

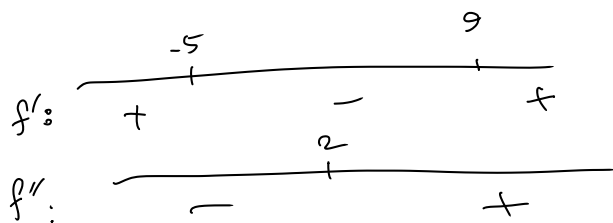
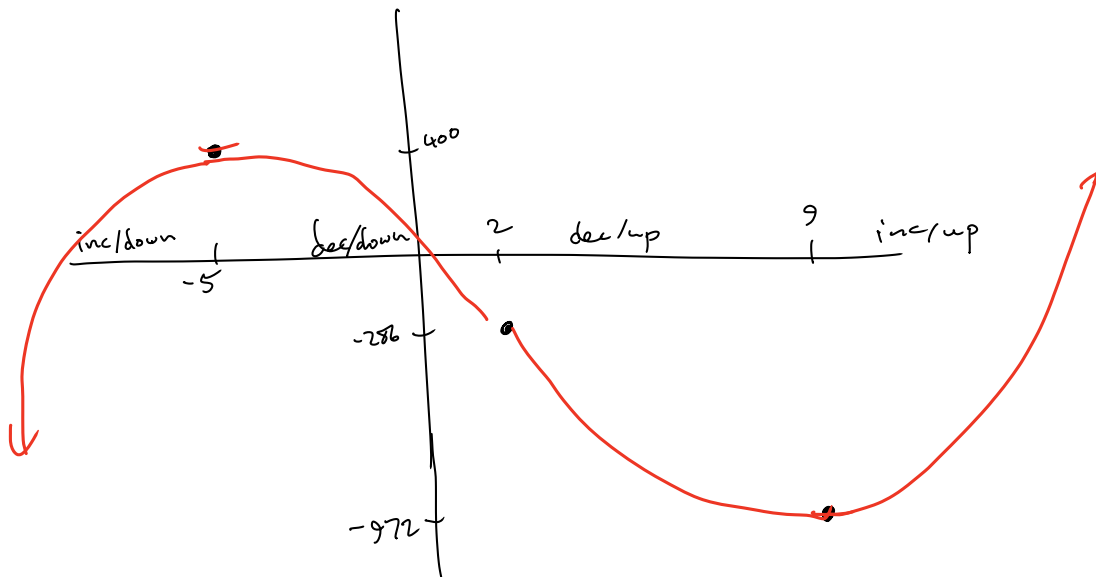
$$\text{crits: } -5, 9$$

$$\text{infl: } 2$$

$$f(-5) = (-5)^3 - 6(-5)^2 - 135(-5) = 400$$

$$f(9) = -972$$

$$f(2) = -286$$

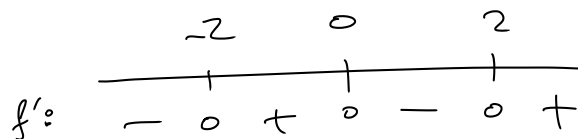


$$f(x) = \frac{1}{2}x^4 - 4x^2 + 3 \quad \text{sketch the graph.}$$

$$f'(x) = 2x^3 - 8x$$

$$= 2x(x^2 - 4) = 2x(x+2)(x-2)$$

$$x=0, x=-2, x=2$$



$$f''(x) = 6x^2 - 8 = \underline{2(3x^2 - 4)}$$

$$f'' = 0: \quad 3x^2 - 4 = 0$$

$$3x^2 = 4$$

$$x^2 = 4/3$$

$$x = \pm \sqrt{4/3} = \pm 1.15 \dots$$



$$f''(-2) = 2(3(-2)^2 - 4)$$

$$= 2(12 - 4)$$

y -values:

$$f(-2) = -5$$

$$f(2) = -5$$

$$f(0) = 3$$

$$f(-\sqrt{4/3}) = -1.44$$

$$f(\sqrt{4/3}) = -1.44$$

