A hard curve-sketch!

$$f(x) = 6x^{2/3} - 4x$$

inc/dec

$$f'(x) = 4x^{-1/3} - 4$$

$$= 4(x^{-1/3} - 1)$$

$$= 4\left(\frac{1}{\sqrt[3]{x}} - 1\right)$$

f' DNE

denom = 0

$\sqrt[3]{x} = 0$

$\boxed{x = 0}$

$f' = 0$

$\boxed{4\left(\frac{1}{\sqrt[3]{x}} - 1\right)} = 0$

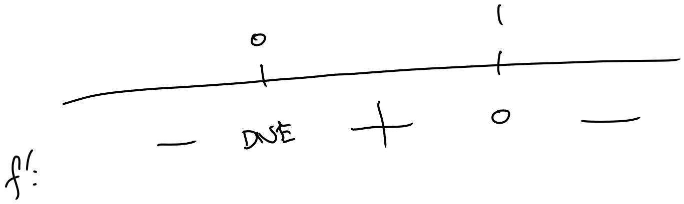
$\frac{1}{\sqrt[3]{x}} - 1 = 0$

$\frac{1}{\sqrt[3]{x}} = 1$

$1 = \sqrt[3]{x}$

$\boxed{1 = x}$

$$4\left(\frac{1}{\sqrt[3]{x}} - 1\right)$$



$$f'(-1) = 4\left(\frac{1}{\sqrt[3]{-1}} - 1\right) = 4\left(\frac{1}{-1} - 1\right) = -8$$

$$f'(8) = 4\left(\frac{1}{\sqrt[3]{8}} - 1\right) = 4\left(\frac{1}{2} - 1\right)$$

$+ \quad -$

$$f(1/8) = 4\left(\frac{1}{\sqrt[3]{1/8}} - 1\right) = 4\left(\frac{1}{1/2} - 1\right) = +$$

$f'(x) = 4x^{-1/3} - 4$

$f''(x) = -\frac{4}{3}x^{-4/3} = -\frac{4}{3}\cdot\frac{1}{x^{4/3}}$

f'' DNE: $\boxed{x=0}$

$\underline{f''=0}$: $\frac{-4}{3}\cdot\frac{1}{x^{4/3}} = 0$

$\frac{1}{x^{4/3}} = 0$

$1 = x^{4/3}\cdot 0$

$1 = 0$ so f'' never $= 0$.

no solutions,

0

f'': — DNE — $\qquad -\frac{4}{3}\cdot\frac{1}{x^{4/3}}$

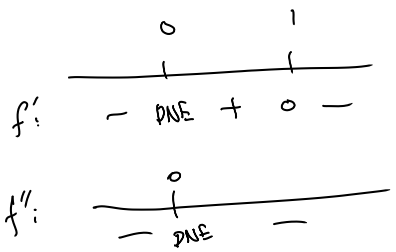
$f''(-1) = \frac{-4}{3}\cdot\frac{1}{(-1)^{4/3}} = \frac{-4}{3}\cdot\frac{1}{1}$

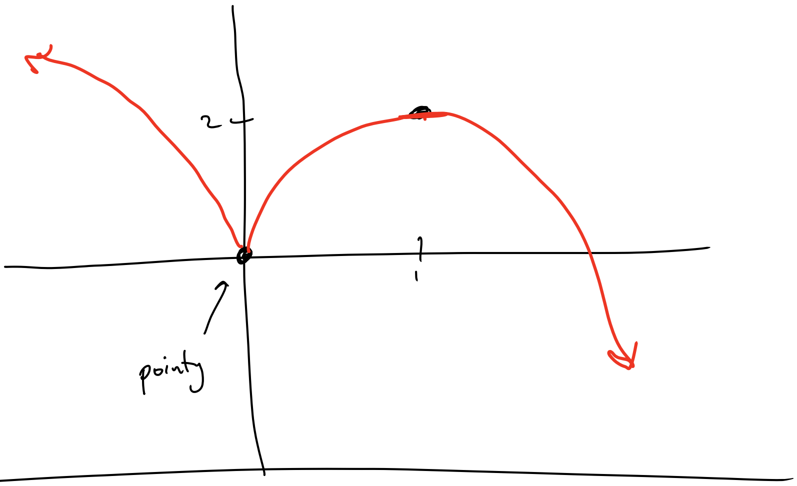
$f''(1) = \frac{-4}{3}\cdot\frac{1}{1^{4/3}} = \frac{-4}{3}$

plot $x=0$, $x=1$ $\quad f(x) = 6x^{2/3} - 4x$

$f(0) = 6\cdot 0^{2/3} - 4\cdot 0 = 0$

$f(1) = 6\cdot 1^{2/3} - 4\cdot 1 = 6-4 = 2$





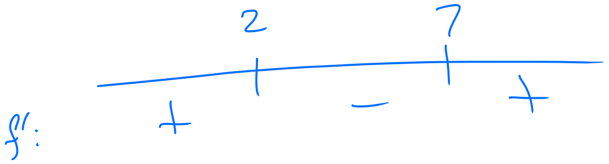
The Second Derivative Test

Another way to identify a local min/max

First Deriv. Test:

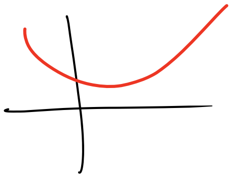
to tell if it's min vs max:

Find critical #s, make the inc/dec chart

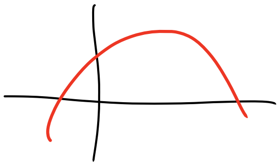


2nd Deriv test answers the same question,
but requires no chart!

f'' positive means this shape:



f'' negative – – —



Thm (2nd Derivative test) Let c be a critical # of f.

If $f''(c) > 0$, then c is a local min

If $f''(c) < 0$, then c is a local max.

If $f''(c) = 0$ or $f''(c)$ DNE, then we don't know.

(then we must use 1st deriv. test)

Ex) Use 2nd deriv test to find local extrema for $f(x) = 4x^3 + 7x^2 - 10x + 8$.

Find critical #s: $f'(x) = 12x^2 + 14x - 10$

$f' = 0$: $12x^2 + 14x - 10 = 0$

$2(6x^2 + 7x - 5) = 0$

$2(2x - 1)(3x + 5) = 0$

$2x - 1 = 0$ $3x + 5 = 0$

$\boxed{x = 1/2}$ $\boxed{x = -5/3}$

plug these into f''!

$f''(x) = 24x + 14$

$f''(\tfrac{1}{2}) = 24 \cdot \tfrac{1}{2} + 14$ so $x = \tfrac{1}{2}$ is a local min

$= 12 + 14 = +$

$f''(-\tfrac{5}{3}) = 24 \cdot -\tfrac{5}{3} + 14$ so $x = -\tfrac{5}{3}$ is a local max

$= 8 \cdot -5 + 14 = -$

Ex) $f(x) = x + \dfrac{4}{x^2} = x + 4x^{-2}$

$f'(x) = 1 - 8x^{-3}$

$f''(x) = 24x^{-4}$

critical #s $f' = 0$: $1 - 8x^{-3} = 0$

$1 - \dfrac{8}{x^3} = 0$

$1 = \dfrac{8}{x^3}$ $x^3 = 8$

$\boxed{x = 2}$

f' DNE $1 - \dfrac{8}{x^3} \leftarrow \boxed{x=0}$

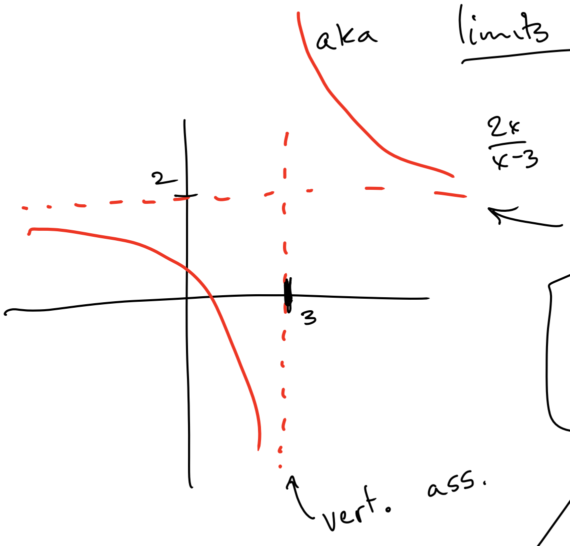
$f(x)$ DNE when $x=0$, so $x=0$ is not a min or max

$f''(2) = 24 \cdot 2^{-4} = \dfrac{24}{2^4} = +$

so $x = 2$ is a local min.

Horizontal Asymptotes

aka limits at ∞.



as x increases forever, f(x) approaches 2.

also written:

$$\lim_{x\to\infty} \frac{2x}{x-3} = 2$$

"limit at ∞"

$$\lim_{x\to-\infty} \frac{2x}{x-3} = 2$$

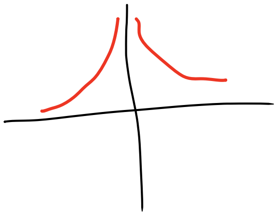
"limit at −∞"

different from "infinite limit"

which is

$$\lim_{x\to 0} \frac{1}{x^2} = \infty$$

limit is infinite



Basic facts abt limits at ∞:

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

in fact, $\displaystyle\lim_{x \to \infty} \frac{k}{x^r} = 0$ if $r > 0$

and $\displaystyle\lim_{x \to -\infty} \frac{k}{x^r} = 0$ if r is a positive integer.