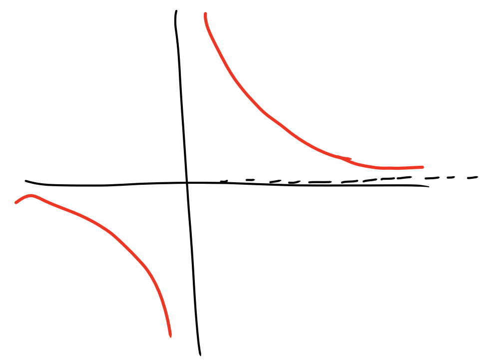
Limits at ∞

(Horz. 'totes)

$$\lim_{x\to\infty} \frac{1}{x} = 0$$



$$\lim_{x\to\infty} \frac{K}{x^r} = 0$$ for any k & $r > 0$

$$\lim_{x\to-\infty} \frac{K}{x^r} = 0$$ for any K and r a positive integer

For more complicated algebraic functions fancy trick: divide everything by the biggest power of x.

$$\lim_{x\to\infty} \frac{3x^2-7x+5}{4x^2+x+1}$$ divide all by x^2

$$= \lim_{x\to\infty} \frac{\frac{3\cancel{x^2}}{\cancel{x^2}} - \frac{7x}{x^2} + \frac{5}{x^2}}{\frac{4\cancel{x^2}}{\cancel{x^2}} + \frac{x}{x^2} + \frac{1}{x^2}} = \lim_{x\to\infty} \frac{3 - \frac{7}{x} + \frac{5}{x^2}}{4 + \frac{1}{x} + \frac{1}{x^2}}$$

$$= \frac{3 - 0 + 0}{4 + 0 + 0} = \frac{3}{4}$$

Also $\lim\limits_{x \to -\infty} \dfrac{3x^2 - 7x + 5}{4x^2 + x + 1} = 3/4$

With radicals, more fancy:

$$\lim_{x \to \infty} \frac{5x^4 + \sqrt{2 + 4x^{10}}}{6x^5 + 7x^2 - 1}$$

x^{10} inside $\sqrt{\ }$ counts like x^5.

"biggest power" is x^5.

$$= \lim_{x \to \infty} \frac{\frac{5x^4}{x^5} + \frac{1}{x^5}\sqrt{2 + 4x^{10}}}{\frac{6x^5}{x^5} + \frac{7x^2}{x^5} - \frac{1}{x^5}}$$

$$= \lim_{x \to \infty} \frac{\frac{5x^4}{x^5} + \sqrt{\frac{2}{x^{10}} + \frac{4x^{10}}{x^{10}}}}{\frac{6x^5}{x^5} + \frac{7x^2}{x^5} - \frac{1}{x^5}}$$

$$= \lim_{x \to \infty} \frac{\frac{5}{x} + \sqrt{\frac{2}{x} + 4}}{6 + \frac{7}{x^3} - \frac{1}{x^5}} = \frac{0 + \sqrt{0 + 4}}{6 + 0 - 0}$$

$$= \frac{\sqrt{4}}{6} = \frac{2}{6} = 1/3$$

we used $\boxed{x\sqrt{y} = \sqrt{x^2 y}}$

this is true only when x is positive.

When x is negative,

$x\sqrt{y}$ will be negative

$\sqrt{x^2 y}$ is positive

when x is negative, $\boxed{x\sqrt{y} = -\sqrt{x^2 y}}$

This often matters in $\lim\limits_{x\to\infty}$ vs $\lim\limits_{x\to-\infty}$

So in a $\lim\limits_{x\to\infty}$, we can do $x\sqrt{y} = \sqrt{x^2 y}$

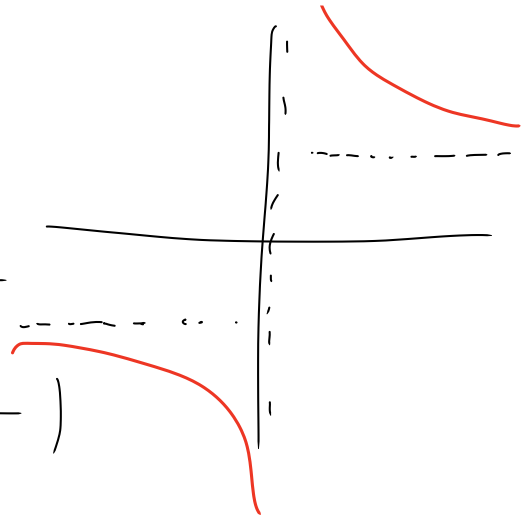
in $\lim\limits_{x\to-\infty}$, it must be $x\sqrt{y} = -\sqrt{x^2 y}$

Ex1 $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2+4}}{x}$ div. by x'

$$= \lim_{x\to\infty} \frac{\frac{1}{x}\sqrt{x^2+4}}{\frac{x}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^2}\cdot x^2 + \frac{1}{x^2}\cdot 4}}{1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1 + 4/x^2}}{1} = \frac{\sqrt{1}}{1} = 1$$

$$\lim_{x\to -\infty} \frac{\sqrt{x^2+4}}{x} = \lim_{x\to -\infty} \frac{\frac{1}{x}\sqrt{x^2+4}}{x/x}$$

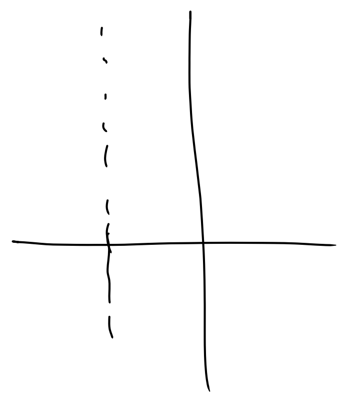


$$= \lim_{x\to -\infty} \frac{-\sqrt{x^2/x^2 + 4/x^2}}{1}$$

$$= \frac{-\sqrt{1+0}}{1} = -1$$

Find any vert/horz 'totes of

$$\frac{\sqrt{5x^2+7x}}{10x+20}$$



<u>vert 'totes:</u> set denom $=0$, solv for x

$$10x+20=0$$

$$10x=-20$$

$$x=-2$$

<u>horz 'totes.</u>

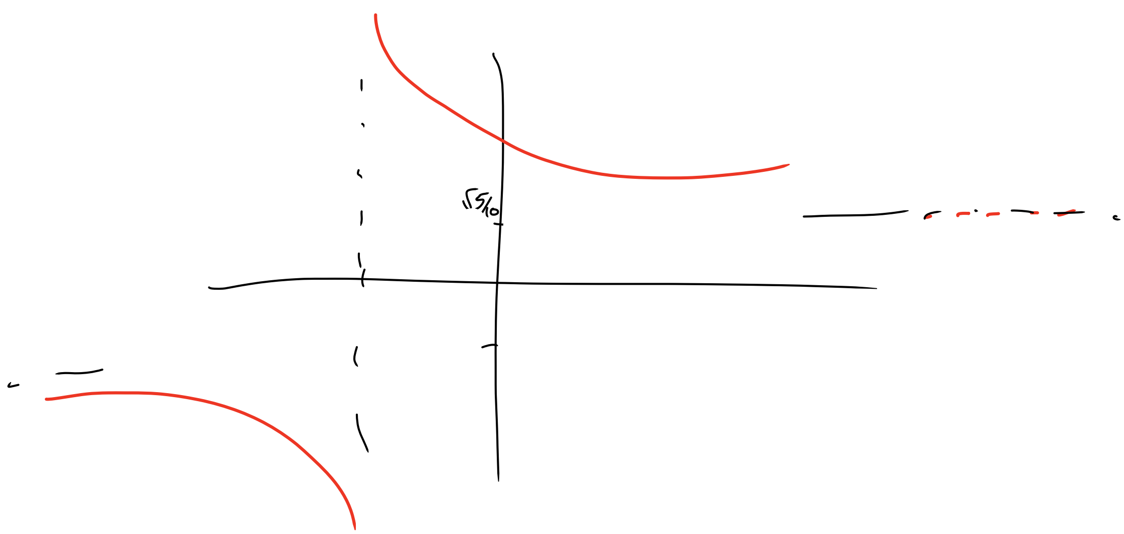
$$\lim_{x\to\infty} \frac{\sqrt{5x^2+7x}}{10x+20}$$ div by x

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{5x^2}{x^2}+\frac{7x}{x^2}}}{\frac{10x}{x}+\frac{20}{x}} = \lim_{x\to\infty} \frac{\sqrt{5+\cancel{\frac{7}{x}}}}{10+\cancel{\frac{20}{x}}}$$

$$= \sqrt{5}/10$$

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2+7x}}{10x+20} \qquad \text{div by } x$$

$$= \lim_{x\to\infty} \frac{-\sqrt{\frac{5x^2}{x^2}+\frac{7x}{x^2}}}{\frac{10x}{x}+\frac{20}{x}} = -\frac{\sqrt{5}}{10}$$



You:

$$\frac{3x+\sqrt{5x+x^2}}{x-5} \qquad \text{vert: } x=5$$

$$\lim_{x\to\infty} \frac{\frac{3x}{x}+\sqrt{\frac{5x}{x^2}+\frac{x^2}{x^2}}}{\frac{x}{x}-\frac{5}{x}} = \lim_{x\to\infty} \frac{3+\sqrt{0+1}}{1-0} = \frac{4}{1} = 4$$

$$\lim_{x\to-\infty} \frac{\frac{3x}{x}-\sqrt{\frac{5x}{x^2}+\frac{x^2}{x^2}}}{\frac{x}{x}-\frac{5}{x}} = \frac{3-\sqrt{1}}{1} = 2$$

"limit at ∞" is like $\lim_{x \to \infty}$

"infinite limit" is like $\lim_{x \to 0} \frac{1}{x^2} = \infty$

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infinite limit at ∞

$$\lim_{x \to \infty} x^2 + 3 = \infty$$

∞ here   also here.

$$\lim_{x \to -\infty} x^2 + 3 = \infty$$

$$\lim_{x \to -\infty} x^3 + 3 = -\infty$$

Have to think it through

$$\lim_{x \to \infty} x^5 = \infty$$

$$\lim_{x \to -\infty} x^5 = -\infty$$

$$\lim_{x \to \infty} x^2 - x = \cancel{\infty - \infty}$$

not zero!

$\downarrow$

$$\lim_{x \to \infty} x(x-1) = \infty$$

Sketching with asymptotes

we need to look asymptotes when we see a denominator.

Ex) Sketch the curve of $f(x) = \dfrac{2x^2+1}{x^2-1}$

'totes vert: $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

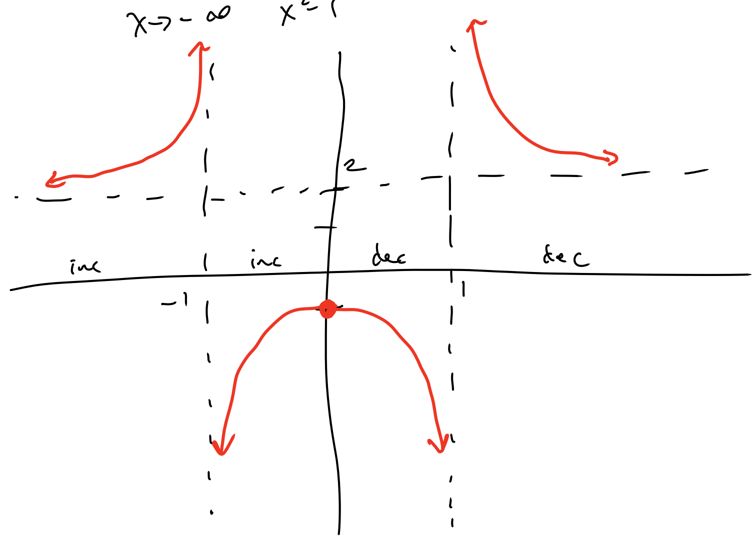
horz: $\displaystyle \lim_{x \to \infty} \frac{2x^2+1}{x^2-1} = \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{2}{1} = 2$

$\displaystyle \lim_{x \to -\infty} \frac{2x^2+1}{x^2-1} = 2$

cnt #s y-vals:

$f(0) = \dfrac{2 \cdot 0^2+1}{0^2-1}$

$= \dfrac{1}{-1} = -1$



Critical #s

$$f(x) = \frac{2x^2+1}{x^2-1}$$

$$f'(x) = \frac{(x^2-1)\cdot 4x - (2x^2+1)\cdot 2x}{(x^2-1)^2}$$

$$= \frac{\cancel{4x^3} - 4x - (\cancel{4x^3} + 2x)}{(x^2-1)^2}$$

$$\boxed{f'(x) = \frac{-6x}{(x^2-1)^2}}$$

$f'=0$: $\boxed{\frac{-6x}{(x^2-1)^2}} = 0$ $\qquad$ f' DNE: $x = +1$, $x = -1$

$$-6x = 0$$

$$x = 0$$

inc/dec

$$\begin{array}{ccccccccc} & & -1 & & 0 & & 1 & \\ f' & + & \text{DNE} & + & 0 & - & \text{DNE} & - \end{array}$$

$f'(-2) = \frac{-6\cdot(-2)}{(\ \ )^2} = \frac{+}{+}$ $\qquad$ $f'(2) = \frac{-6\cdot 2}{(\ \ )^2} = -$

$f'(-1/2) = \frac{-6(-1/2)}{(\ \ )^2} = \frac{+}{+}$ $\qquad$ $f'(1/2) = \frac{-6\cdot 1/2}{+} = \frac{-}{+} = -$

Optimization Problems

Find the value of ~~~~ which maximizes ~~~

(or minimizes)

Ex1 Say when I make x jars of syrup, my profits obey:

$$f(x) = -6 - x^2 + 40x.$$

I can make anywhere from 0 to 100 jars.

How many should I make to maximize profit?

This is an absolute extrema problem, interval of x values is: $[0, 100]$.