Optimization Problems

Ex1 When I make x jars, my profit is

$f(x) = -6 - x^2 + 40x.$

I can make up to 100 jars.

How many should I make to maximize profit?

We're looking for the absolute max of $f(x)$ on the interval: $[0, 100]$

Find crit #s, plug in values at crit #s and interval endpts.

$f(x) = -6 - x^2 + 40x.$

$f'(x) = -2x + 40$

$f' = 0$ $\quad -2x + 40 = 0$

$-2x = -40$

$x = 20$

plug in to $f(x)$

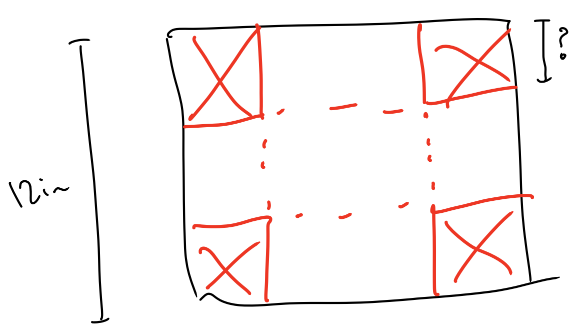
$f(0) = -6 - 0^2 + 40 \cdot 0 = -6$

$f(100) = -6 - 100^2 + 40 \cdot 100 = -6006$

$f(20) = -6 - 20^2 + 40 \cdot 20 = 392 \leftarrow$ max

I should make 20 jars.

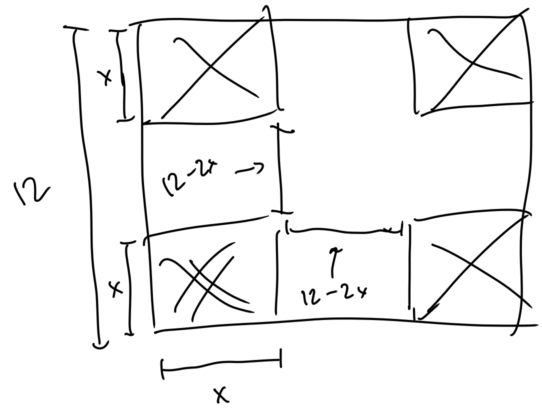
Ex1 I'm making a cardboard box with no top, starting with a 12 in square, cutting out the corners



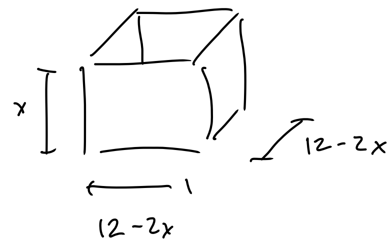
How big should the cutouts be to maximize the volume of the box?

- Figure out what needs to be maximized / minimized
- Write a formula for it in terms of one variable. $f(x)$
- Decide the allowable interval of x values
- Do it!

We're maximizing the volume of the box.



folded box:



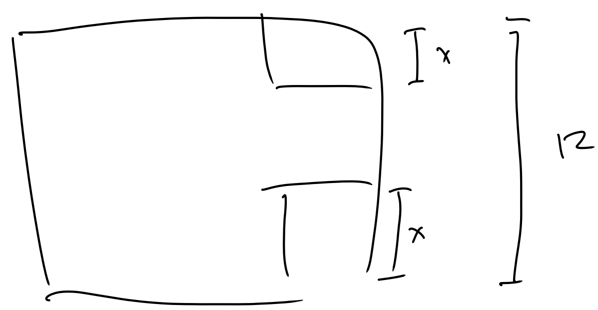
Volume is

$$V(x) = x(12-2x)(12-2x)$$

$$= x(144 - 48x + 4x^2)$$

$$\boxed{V(x) = 144x - 48x^2 + 4x^3}$$

<u>interval</u>:



smallest possible x: $x=0$

biggest : $x=6$

interval $[0, 6]$

- Do it!

$$V'(x) = 144 - 96x + 12x^2$$
$$= 12(x^2 - 8x + 12)$$
$$= 12(x-6)(x-2)$$

$V'(x) = 0$: $x = 6$, $x = 2$

plug in: $\boxed{V(x) = 144x - 48x^2 + 4x^3}$

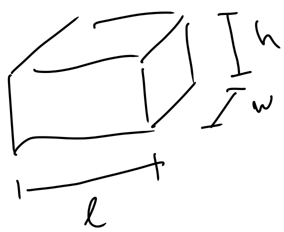
$$V(0) = 0$$
$$V(6) = 144 \cdot 6 - 48 \cdot 6^2 + 4 \cdot 6^3 = 0$$
$$V(2) = 144 \cdot 2 - 48 \cdot 2^2 + 4 \cdot 2^3 = 128$$

max

Cutouts with length 2 in will maximize the volume.

I want to send a 2 in. high box by US media mail

"maximum combined linear distance must not exceed 108 in"



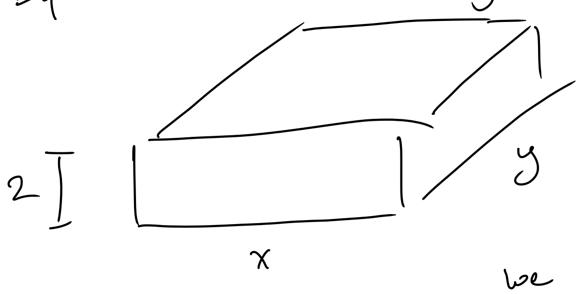
$l + w + h$

What's the biggest volume box I can send?

& which dimensions make the biggest volume?

We'll maximize the volume.

Equation for vol using 1 variable:



$V = 2 \cdot x \cdot y$

we need to express y in terms of x.

We'll also have $2 + x + y = 108$

solve for y: $x+y = 106$

$y = 106 - x$

So $V = 2x(106-x)$

$\boxed{V(x) = 212x - 2x^2}$

$x+y=106$

interval:

smallest x: 0

biggest x: 106

interval is $[0, 106]$

Do it!

$V'(x) = 212 - 4x$

$V'=0$: $212 - 4x = 0$

$4x = 212$

$x = 53$

plug in:

$V(0) = 212 \cdot 0 - 2 \cdot 0^2 = 0$

$V(106) = 0$

$V(53) = 5618$ ← max!

Max possible volume is 5618 in^3

This volume happens when dims are

$2 \times 53 \times 53$ in