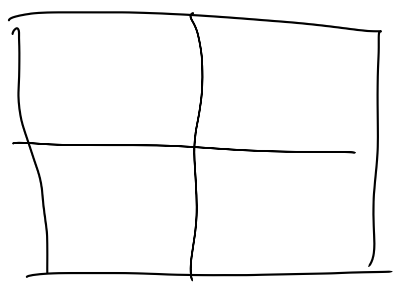
Optimization

- Decide what should be maximized/minimized
- Write it as a function of a single variable.
- Find the interval of allowable x-values.
- Do abs max/min.

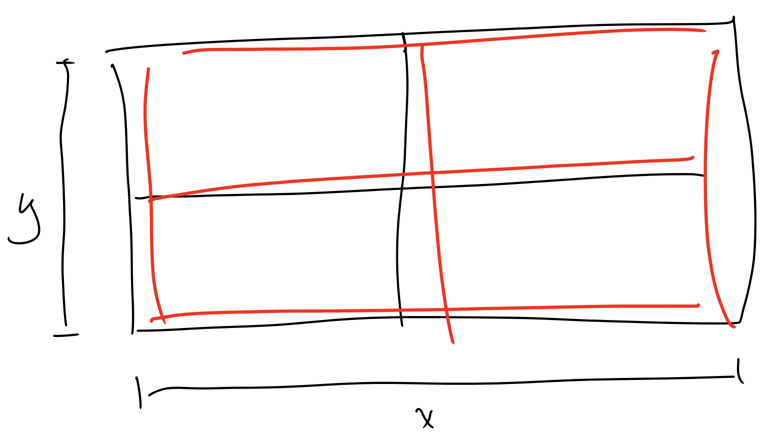
I'm building a fenced-in area:



I have only 300 ft of fencing

What's the biggest area I can enclose,
and which dimensions achieve that area?

We'll maximize the area



Total area:

$$A = xy$$

we need to write y interms of x.

300 ft of fencing:

$$3x + 3y = 300$$

$$x + y = 100$$

$$y = 100 - x$$

So $$A(x) = x(100 - x)$$

$$\boxed{A(x) = 100x - x^2}$$

interval: $x + y = 100$

smallest x is $x = 0$

biggest x is $x = 100$ (when $y = 0$)

interval is $[0, 100]$

Abs max: $$A(x) = 100x - x^2$$

$$A'(x) = 100 - 2x$$

$$100 - 2x = 0$$

$$2x = 100$$

$$x = 50$$

$A(x) = 100x - x^2$

Plug in: $$A(0) = 0$$

$A(100) = 0$

$A(50) = 100 \cdot 50 - 50^2$

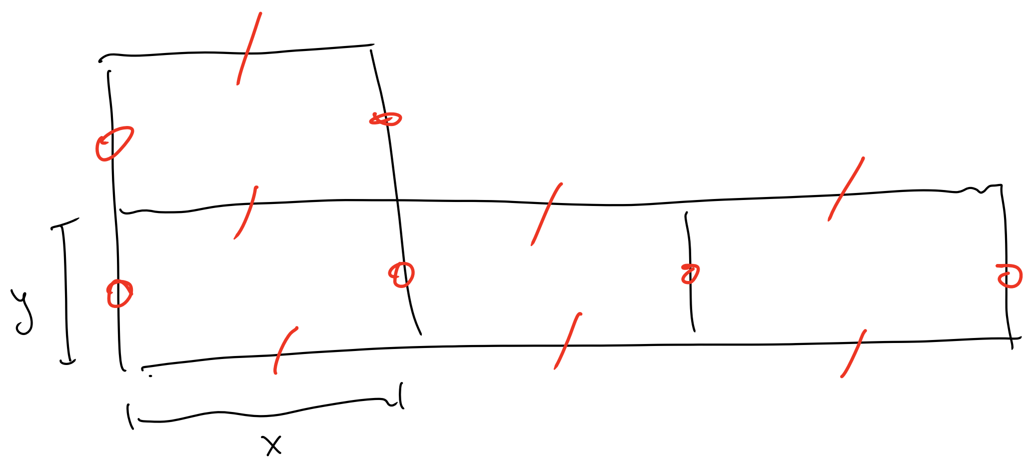
$= 2500$

Biggest area is 2500 ft^2

This happens when $x = 50$ ft

and $y = 100 - x$

$= 50$ ft.



300 ft of fencing

$\downarrow$

$A = 4xy$

$300 = 7x + 6y$

$\downarrow$

$6y = 300 - 7x$

$y = 50 - \frac{7}{6}x$

$A(x) = 4x\left(50 - \frac{7}{6}x\right)$

$$\boxed{A(x) = 200x - \frac{14}{3}x^2}$$

$300 = 7x + 6y$

smallest $x = 0$

biggest x will be when $y = 0$

$300 = 7x$

$x = 300/7$

interval is $[0, 300/7]$

$A(x) = 200x - \frac{14}{3}x^2$

$A'(x) = 200 - \frac{28}{3}x = 0$

$200 = \frac{28}{3}x$

$9.\bar{3}\,x = 200$

$$\boxed{x = 21.42}$$

plug in:

$A(0) = 0$

$A(300/7) = 0$

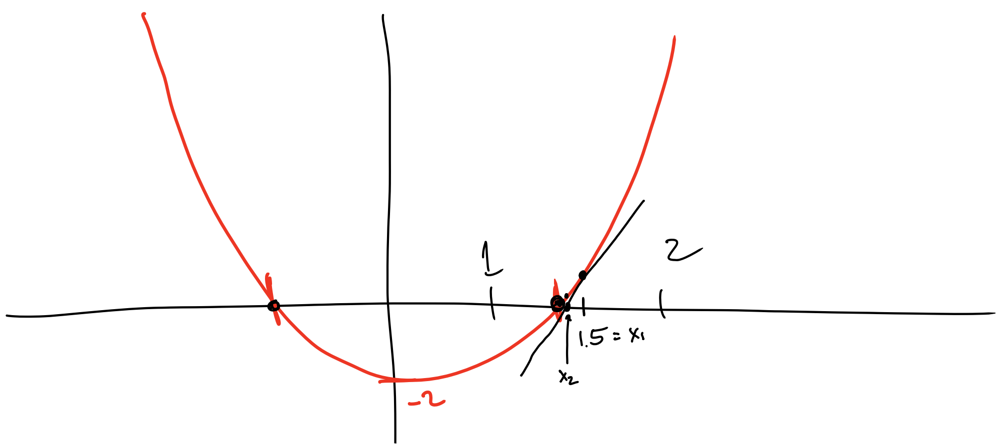
$A(21.42) = 2142 \text{ ft}^2 \leftarrow$ max

Newton's Method

Another method for approximating stuff,
used to approximate x values
where $f(x) = 0$ "The roots of $f(x)$"

Let's find a root of $f(x) = x^2 - 2$

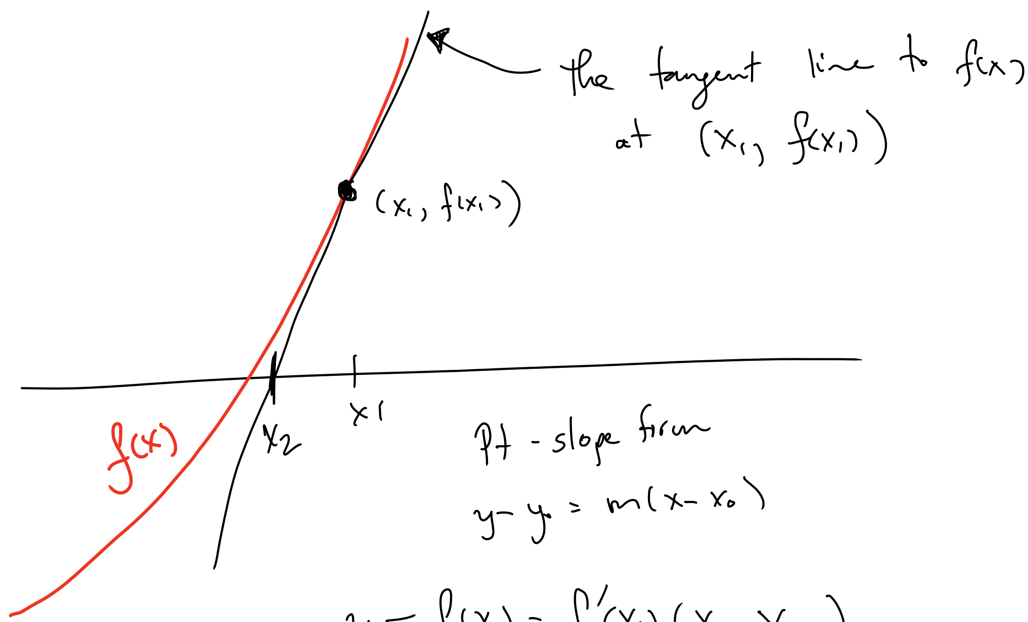
we finding where $x^2 - 2 = 0$
i.e. x-intercepts.



Concept: Start with a guess, then
zoom in & follow the tangent line.

guess $x_1 = 1.5$, calculate x_2.

What exactly is x_2? (in terms of x_1?)



Pt - slope form

$$y - y_0 = m(x - x_0)$$

$$y - f(x_1) = f'(x_1)(x - x_1)$$

x_2 is the x-intercept of this line.

(set $y = 0$, solve for x)

$$-f(x_1) = f'(x_1)(x - x_1)$$

$$-\frac{f(x_1)}{f'(x_1)} = x - x_1$$

$$x = \boxed{x_1 - \frac{f(x_1)}{f'(x_1)}}$$ This is x_2

Generally, we keep on plugging in to get better approximations

guess x_1

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

etc.

Generally:

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$