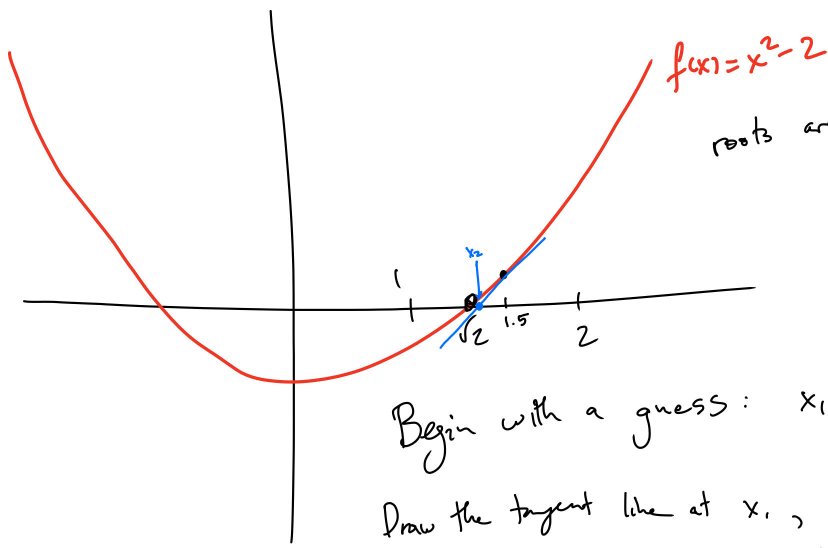
Newton's Method

Estimates the roots of an equation.

$f(x) = 0$.



roots are $\pm\sqrt{2}$

$x_1 = 1.5$

Begin with a guess: x_1

Draw the tangent line at x_1,

x_2 is the x-intercept of that tangent line.

An easy formula:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

generally,

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

let's try to estimate $\sqrt{2}$ using Newt's meth with first guess $x_1 = 1.5$.

Choose the function: $x^2 - 2 = 0$

$f(x) = x^2 - 2$, $x_1 = 1.5$

$f'(x) = 2x$

$x_1 = 1.5$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

actually,

$\sqrt{2} = 1.414213\ldots$

$x_2 = 1.5 - \frac{1.5^2 - 2}{2 \cdot 1.5} = 1.41666\ldots$

$x_3 = 1.41\overline{6} - \frac{1.41\overline{6} - 2}{2 \cdot 1.41\overline{6}} = 1.414215\ldots$

Estimate $\sqrt[10]{100}$ with first guess $x_1 = 2$.

The function: need some f where $f(\sqrt[10]{100}) = 0$

$f(x) = x^{10} - 100$, $x_1 = 2$

$f'(x) = 10x^9$

$$\frac{f(x)}{f'(x)} = \frac{x^{10} - 100}{10x^9} = \frac{x}{10} - \frac{10}{x^9}$$

$x_1 = 2$

$$x_2 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{2^{10} - 100}{10 \cdot 2^9}$$

$$= 1.819$$

$x_4 = 1.607 \ldots$

$x_5 = 1.586229 \ldots$

$x_6 = 1.584898245 \ldots$

$x_7 = 1.584893193 \ldots$

"Estimate to within the nearest thousandth"

1.584

- Estimate $\sqrt[3]{9}$ to the nearest thousandth

- Find a root of
  $4x^3 - 5x^2 + 10$ in the interval $[-4, 0]$.

  (go to x_3)

$x^3 - 9 = 0$ $\quad x_1 = 2$

$f(x) = x^3 - 9$

$f'(x) = 3x^2$

$x_1 = 2$

$x_2 = 2 - \dfrac{2^3 - 9}{3 \cdot 2^2} =$

$\vdots$

$2.0800 \ldots$

$f(x) = 4x^3 - 5x^2 + 10$ on $[-4, 0]$

start with $x_1 = -2$ ← middle of the interval.

$f'(x) = 12x^2 - 10x$

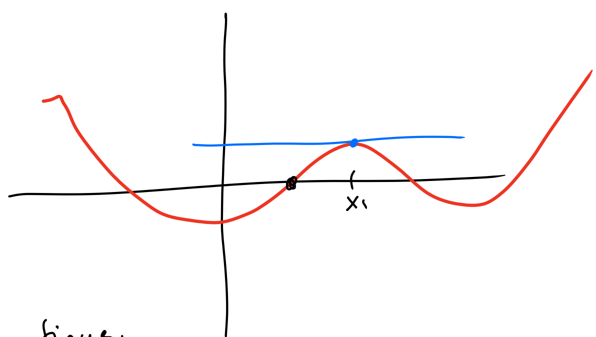
$$x_2 = -2 - \frac{4(-2)^3 - 5(-2)^2 + 10}{12(-2)^2 - 10(-2)} = \dots$$

$x_3 =$

Sometimes Newton's Method fails

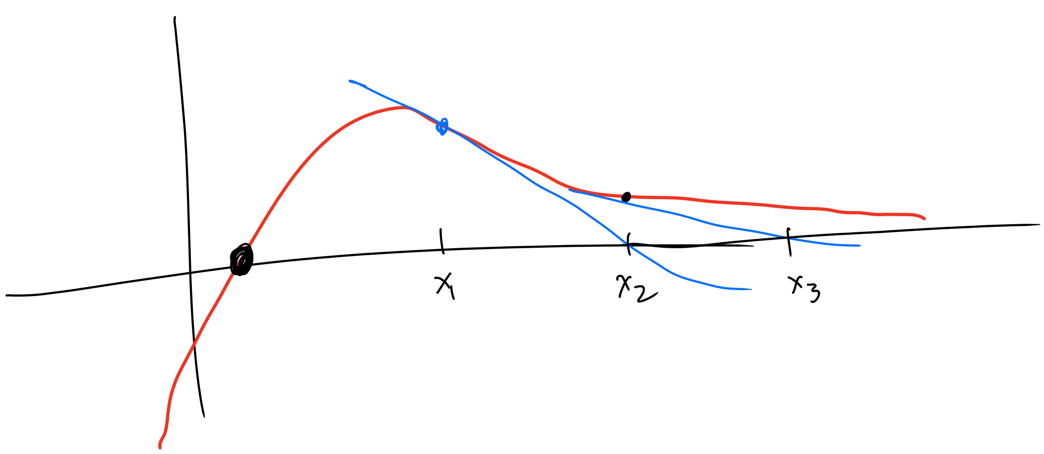
1 if $f'(x_1) = 0$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$



if $f'(x_1) = 0$, we can't continue,
but we could adjust our guess so that $f'(x_1) \neq 0$.

2



Here, the x_n do not approach the root!

Here, "Newt's method does not converge"

The Antiderivative

Given $f(x)$, an antiderivative of f is some function $F(x)$ with:

$$\frac{d}{dx}F(x) = f(x).$$

"deriv. of F is f."

means same thing as: "antideriv of f is F"

The deriv of x^2 is $2x$.

so, an antiderivative of $2x$ is x^2

another antideriv of $2x$ is x^2+1

We can always change the antideriv. by adding a constant, and we'll get another antideriv.

So $2x$ has antiderivs like x^2, x^2+1, x^2+18, ⋮ } ← these are "particular antiderivs"

we say x^2+C is the "general antiderivative"

$f(x)$	$F(x)$
$2x$	x^2+C
$3x^2$	x^3+C
x^2	$\frac{1}{3}x^3+C$
x^n	$\boxed{\frac{1}{n+1}x^{n+1}+C}$

$f(x)$	$F(x)$	
x^n	$\frac{1}{n+1}x^{n+1}+C$	← works for any n except $n=-1$
k	$kx+C$	
$\cos x$	$\sin x+C$	
$\sin x$	$-\cos x+C$	
$\sec^2 x$	$\tan x+C$	

etc.

every deriv formula can be rewritten as an antideriv. formula

What if we have stuff added together or multiplied by constants?

Thm Let f have antideriv F, and g have antideriv G, then

- The antideriv of $f+g$ is $F+G$.
- The antideriv of kf is kF

Find antideriv:

$f(x) = 5x^2 - 7x^4$

$F(x) = 5 \cdot \frac{1}{3}x^3 - 7 \cdot \frac{1}{5}x^5 + C$

$f(x) = \frac{4}{x^2} + \sqrt{x}$ rewrite as powers of x

$f(x) = 4x^{-2} + x^{1/2}$

so $F(x) = 4 \cdot \frac{1}{-1} x^{-1} + \frac{1}{3/2} x^{3/2} + C$

$= -4x^{-1} + \frac{2}{3} x^{3/2} + C$