Curve sketching

Sketch : $f(x) = 2x^3 - 18x^2 - 6$

$f'(x) = 6x^2 - 36x$

$\boxed{f'(x) = 6x(x-6)}$

$\underline{f'=0:}$ $6x(x-6) = 0$

$6x = 0$ $\quad$ $x - 6 = 0$

$\underline{x = 0}$ $\quad$ $\underline{x = 6}$

		0		6	
f':	+	0	−	0	+

$f'(-1) = 6(-1)(-1-6)$
$+ \cdot - \cdot - = +$

$f'(1) = 6(1)(1-6)$
$+ \; + \; -$

$f'(7) = 6(7)(7-6) = +$

$f''(x) = 12x - 36$

$\underline{f''=0}$ $\quad 12x - 36 = 0$

$12x = 36$

$x = 3$

		3	
f'':	−	0	+

$f''(0) = 12 \cdot 0 - 36 = -36$

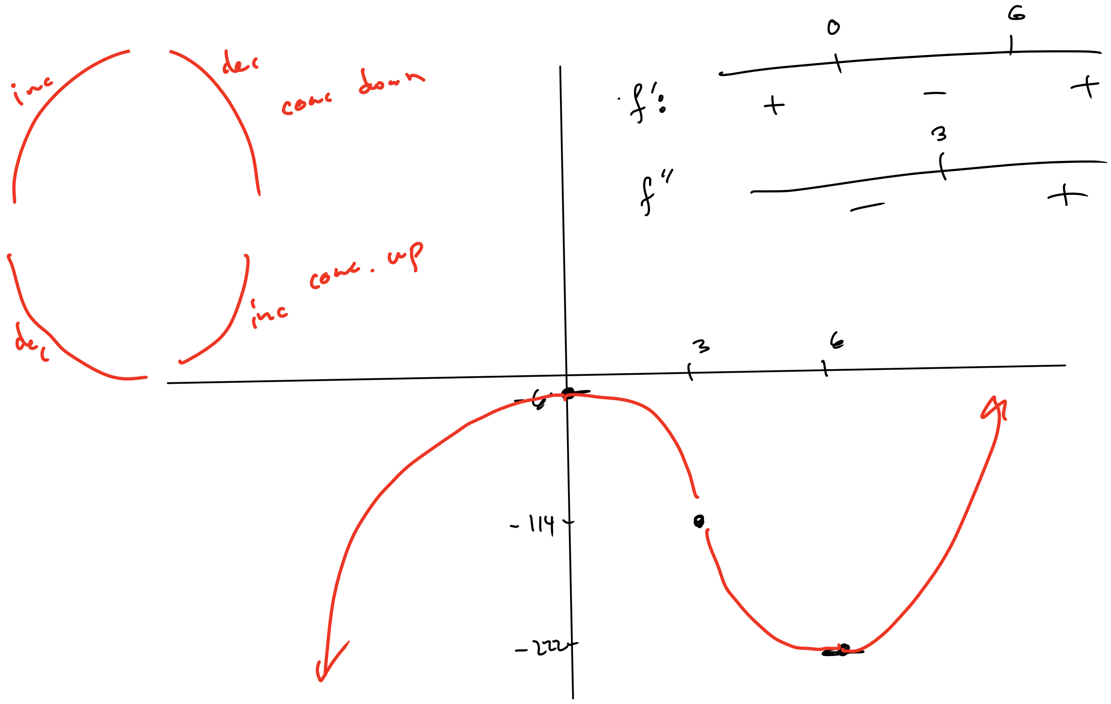
$f''(4) = 12 \cdot 4 - 36 = +$

$f(x) = 2x^3 - 18x^2 - 6$

$\underline{\text{y-values}}$: $f(0) = 2 \cdot 0^3 - 18 \cdot 0^2 - 6 = -6$

$f(3) = 2 \cdot 3^3 - 18 \cdot 3^2 - 6 = -114$

$$f(6) = 2\cdot 6^3 - 18\cdot 6^2 - 6 = -222$$



Estimate $\sqrt{51}$ with Linear approx.

$$L(x) = f(a) + f'(a)(x-a)$$

a is a nearby point where $f(a)$ is already known.

choose $a=49$

we'll use $x=51$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$L(51) = f(49) + f'(49)\,(51-49)$$

$$= \sqrt{49} + \frac{1}{\cancel{2}\sqrt{49}} \cdot \cancel{2}$$

$$= 7 + \frac{1}{7}$$

A sphere is growing, volume increases at $1\ \text{cm}^3/\text{s}$,
How fast is radius increasing when $r = 10$ cm?

2 quantities changing : volume & radius

formula : $V = \frac{4}{3}\pi r^3$

imp. deriv $\frac{d}{dt}$: $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$

plug & solve : solve for : $\frac{dr}{dt}$

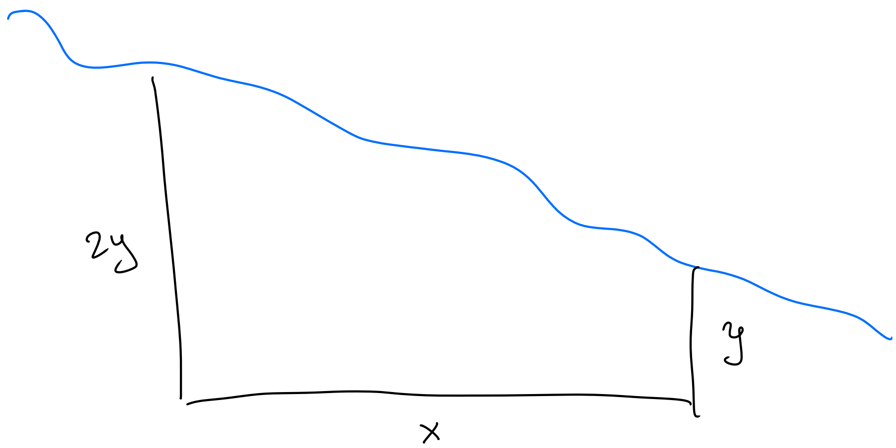
plug $\frac{dV}{dt} = 1$

$r = 10$

$$1 = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} \cdot 10^2 \cdot \frac{dr}{dt}$$

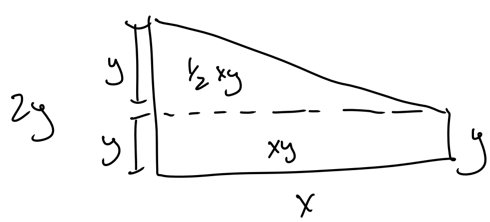
$$1 = 400\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{400\pi} \quad cm/s$$



Which dimensions give the most area, using 300 ft of fence.

Maximize the area.



$$A = xy + \frac{1}{2}xy$$

$$A = \frac{3}{2}xy$$

fence length: $300 = x + 3y$

$$3y = 300 - x$$

$$y = 100 - \frac{1}{3}x$$

So $A(x) = \frac{3}{2}x\left(100 - \frac{1}{3}x\right)$

$$A(x) = 150x - \frac{1}{2}x^2$$

interval of allowable x-values:

$300 = x + 3y$

biggest: $x = 300$

smallest: $x = 0$

Abs max

$$A'(x) = 150 - x$$

$$150 - x = 0$$

$$150 = x$$

y-values:

$$A(0) = 0$$

$$A(300) = 0$$

$$A(150) = 150 \cdot 150 - \frac{1}{2} \cdot 150^2$$

$$= \frac{1}{2} \cdot 150^2 = 11250. \leftarrow \text{max.}$$

$$A(x) = 150x - \frac{1}{2}x^2$$

Most area happens when

$$\boxed{x = 150},$$

$$y = 100 - \frac{1}{3}x$$

$$= 100 - \frac{1}{3} \cdot 150$$

$$= 100 - 50$$

$$\boxed{y = 50}$$