$f(x)$	$F(x)$	
x^n	$\frac{1}{n+1}x^{n+1} + C$	when $n \neq -1$
x^{-1}	$\ln\lvert x\rvert + C$	
$\cos x$	$\sin x + C$	
$\sin x$	$-\cos x + C$	
k	$kx + C$	

For sums & constant multiples, everything adds up nicely

$$f(x) = 8x^5 + 7x^{10} - 2x + 3$$

antidiv 'zi

$$F(x) = 8 \cdot \frac{1}{6}x^6 + 7 \cdot \frac{1}{11}x^{11} - 2 \cdot \frac{1}{2}x^2 + 3x + C$$

Often we need to rewrite first:

we want powers of x.

$$f(x) = \frac{4}{x^2} + \sqrt{x}$$

$$f(x) = 4x^{-2} + x^{1/2}$$

$$F(x) = 4 \cdot \frac{1}{-1}x^{-1} + \frac{1}{3/2}x^{3/2} + C$$

$$= -4x^{-1} + \frac{2}{3}x^{3/2} + C$$

$$\int f(x) = 3\sin x + \frac{8x+x^2}{\sqrt{x}}$$

$$= 3\sin x + \frac{8x+x^2}{x^{1/2}}$$

$$= 3\sin x + (8x^1+x^2)\,x^{-1/2}$$

$$f(x) = 3\sin x + 8x^{1/2} + x^{3/2}$$

$$F(x) = 3\cdot -\cos x + 8\cdot\frac{1}{3/2}x^{3/2} + \frac{1}{5/2}x^{5/2} + C$$

Find the particular antideriv. for

$f(x) = 3\sin x$

which has $F(0) = 2$.

lets us solve for the C

"an initial condition"

First find the general antideriv:

$$F(x) = -3\cos x + C$$

Next plug in $F(0)=2$: $x=0$, set it all $= 2$.

$$-3\cos 0 + C = 2 \quad \text{solve for } C!$$

$$-3\cdot 1 + C = 2$$

$$-3 + C = 2$$

$$C = 5$$

The particular antiderivative is

$$F(x) = -3\cos x + 5$$

My rocket starts at rest, then begins accelerating at $3 \, m/s^2$. Give functions for its velocity & position.

$v(t)$ (velocity) $\quad$ $s(t)$ (position) $\quad$ accel. $= a(t)$.

$v(t)$ is deriv. of $s(t)$

$a(t)$ is deriv of $v(t)$.

So $v(t)$ is antideriv. of $a(t)$

and $s(t)$ is antideriv of $v(t)$

We have $a(t) = 3$

So $v(t)$ is antideriv of $a(t) = 3$, so

$$v(t) = 3t + C$$

Can we solve for C?

plug in $v(0) = 0$ $\quad$ (starts at rest)

plug $t=0$, set it $=0$

$$3 \cdot 0 + C = 0$$

$$C = 0$$

So $\boxed{v(t) = 3t}$

For $s(t)$: $s(t)$ is antiderivative of $v(t)$.

So $s(t) = \frac{3}{2}t^2 + C$

solve for the C: we start at height 0,

so $s(0) = 0$

$\frac{3}{2} \cdot 0^2 + C = 0$

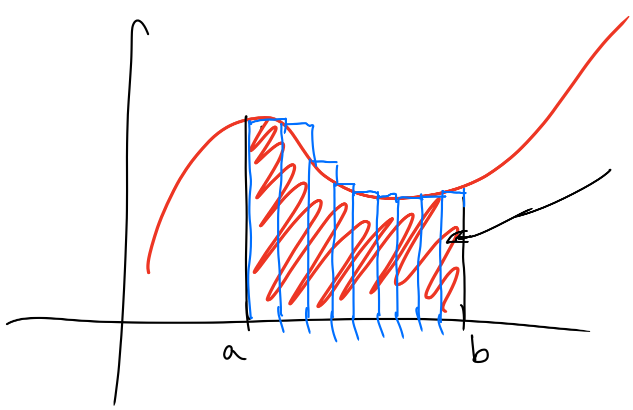
so $C = 0$,

so $\boxed{s(t) = \frac{3}{2}t^2}$

Areas

Seems completely unrelated.

Finding areas of weird shapes.



We'll focus on "the area under the curve" from a to b

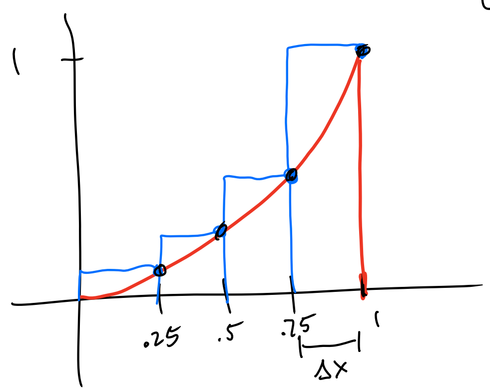
Basic approach: cover the area with little rectangles.

or other shapes.

Then Add up the rectangle areas.

We get an approximation, but, we can get the true area by doing some fancy limits.

For $f(x) = x^2$, on the interval $[0, 1]$



let's make 4 rectangles, with heights impacting the curve on the right side

let's add the rectangle areas!

we'll add up base x height

base each time is $\Delta x = .25$

heights are given by the y-values of .25, .5, .75, & 1.

So heights are $.25^2, .5^2, .75^2, 1^2$

So the rectangle area is:

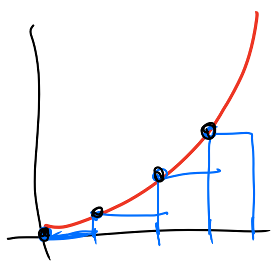
$$\underset{\text{height}}{.25^2} \cdot \underset{\text{base}}{.25} + .5^2 \cdot .25 + .75^2 \cdot .25 + 1^2 \cdot .25$$

$$= .46875.$$

This is "estimate the area using 4 rectangles on the right endpoints"

$R_4 = .46875$ (an overestimate)

Let's try on the left endpoints:



Δx is same. $\Delta x = .25$,

heights are

$0^2, .25^2, .5^2, .75^2$

This approximation is:

$$L_4 = 0^2 \cdot .25 + .25^2 \cdot .25 + .5^2 \cdot .25 + .75^2 \cdot .25$$

$$= .21875$$