$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i\,\Delta x$$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\boxed{\begin{aligned} \sum 1 &= n \\ \sum i &= \frac{n(n+1)}{2} \\ \sum i^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum i^3 &= \left(\frac{n(n+1)}{2}\right)^2 \end{aligned}}$$

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Find $\displaystyle\int_0^4 2x^2+1\,dx$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

$$x_i = a + i\,\Delta x = 0 + i\cdot\frac{4}{n} = \frac{4i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\cdot\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2\left(\frac{4i}{n}\right)^2 + 1\right)\cdot\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2\cdot\frac{16i^2}{n^2} + 1\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{128\,i^2}{n^3} + \frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{128\,i^2}{n^3} + \sum_{i=1}^{n} \frac{4}{n}$$

$$= \lim_{n\to\infty} \frac{128}{n^3} \sum_{i=1}^{n} i^2 + \frac{4}{n} \sum_{i=1}^{n} 1$$

$$= \lim_{n\to\infty} \frac{128}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{4}{\cancel{n}} \cdot \cancel{n}$$

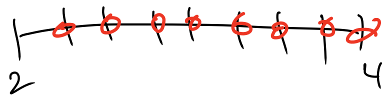
$$= \lim_{n\to\infty} 128 \frac{n(n+1)(2n+1)}{6n^3} + 4$$

$$= 128 \cdot \frac{2}{6} + 4$$

$$= \boxed{128 \cdot \frac{1}{3} + 4}$$

---

For $\int_2^4 3x - 2 \, dx$

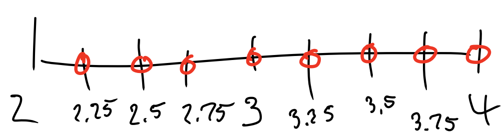



- Find $R_8$ & $L_8$

- Find the integral by doing $\lim_{n\to\infty} R_n$.

---

$R_8$




$\Delta x = \frac{b-a}{n}$

$= \frac{4-2}{8} = \frac{2}{8} = \frac{1}{4}$

$R_8 = .25 f(2.25) + .25 f(2.5) + \ldots + .25 f(4)$

$R_8 = .25\cdot(3\cdot 2.25-2) + .25(3\cdot 2.5-2) + \dots + .25(3\cdot 4-2)$

$L_8 = .25(3\cdot 2-2) + .25(3\cdot 2.25-2) + \dots + .25(3\cdot 3.75-2)$

$\int_2^4 3x-2 \, dx$

$\Delta x = \frac{b-a}{n} = \frac{4-2}{n} = \frac{2}{n}$

$x_i = a + i\Delta x = 2 + i\cdot\frac{2}{n}$

$= 2 + \frac{2i}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2+\frac{2i}{n}\right)\cdot\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3\left(2+\frac{2i}{n}\right)-2\right)\cdot\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(6+\frac{6i}{n}-2\right)\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4+\frac{6i}{n}\right)\frac{2}{n}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8}{n} + \frac{12i}{n^2}$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8}{n} + \sum_{i=1}^{n} \frac{12i}{n^2}$

$= \lim_{n\to\infty} \frac{8}{n}\sum_{i=1}^{n} 1 + \frac{12}{n^2}\sum_{i=1}^{n} i$

$= \lim_{n\to\infty} \frac{8}{\cancel{n}}\cdot\cancel{n} + \frac{12}{n^2}\cdot\frac{n(n+1)}{2}$

$= \lim_{n\to\infty} 8 + 6\cdot\frac{n(n+1)}{n^2}$

$= \quad 8 + 6 \cdot 1 \quad = 14$

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# The Fundamental Theorem of Calculus

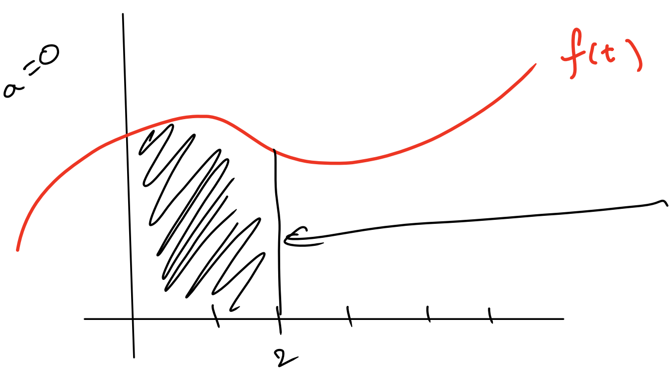
(The trick to do integrals without R. sums)

Two versions: ~~#1~~ FTC #1 is about

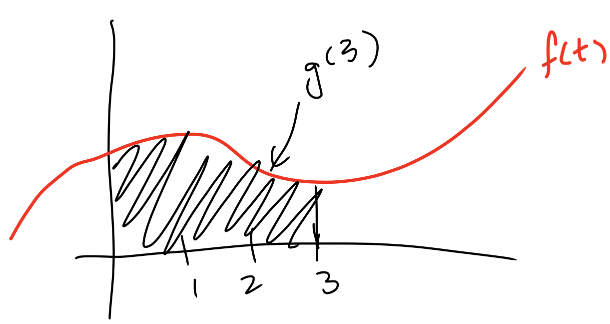
$$g(x) = \int_a^x f(t)\,dt$$

this is the area under $f(t)$ up to $x$.

if $a=0$

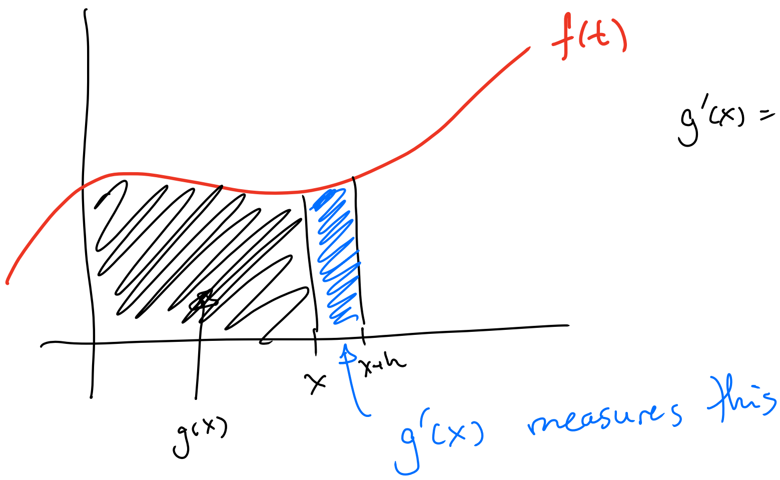



$g(2)$ would be the area up to $x=2$



FTC #1 is about

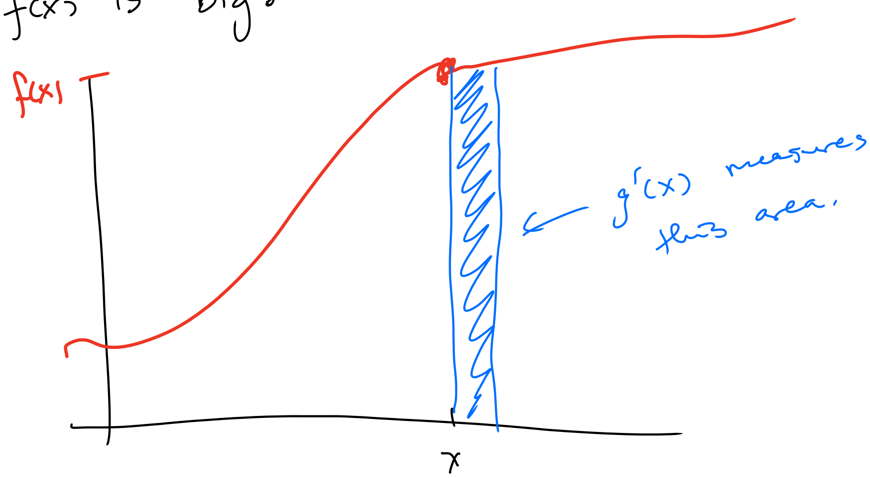
$$\frac{d}{dx} g(x) = \frac{d}{dx}\int_a^x f(t)\,dt$$

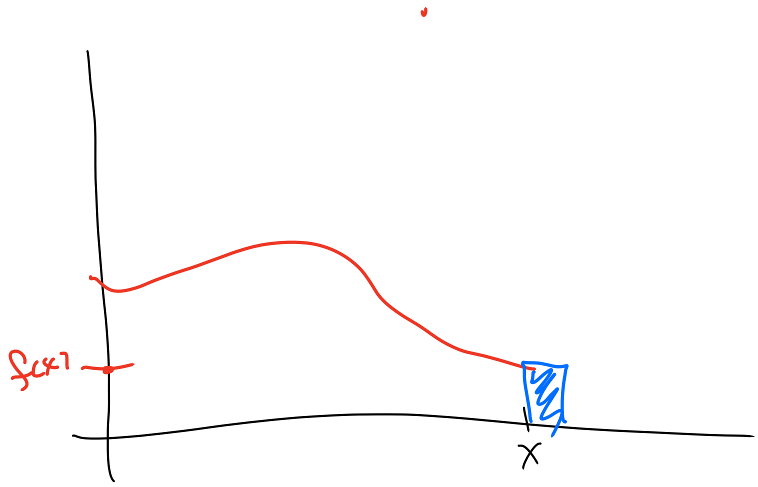



$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

Notice: $g'(x)$ depends a lot on $f(x)$

if $f(x)$ is big:






$g'(x)$ relates to $f(x)$

if $f(x)$ is big, $g'(x)$ is big

if $f(x)$ is small, $g'(x)$ is small

Actually $g'(x) = f(x)$

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$ FTC #1