Indefinite Integral

FTC #1 $\qquad \frac{d}{dx} \int_a^x f(t)\,dt = f(x)$

looks like "$\frac{d}{dx}$ cancels the $\int$"

specifically: $\int_a^x f(t)\,dt$ is an antideriv. of $f(x)$

so we use

$\int f(x)\,dx$ to denote the general antiderivative of $f(x)$.

"the indefinite integral"

$\int_a^b f(x)\,dx$ ← The definite integral.

The area under the curve on $[a,b]$

a number

$\int f(x)\,dx$ ← the indefinite integral

the antiderivative

a function (with $+ C$)

Ex 1 $\int x^3 - 7x^2 + 5 \, dx$

$= \frac{1}{4}x^4 - 7 \cdot \frac{1}{3}x^3 + 5x + C$

No more homework!

Exam Fri Dec 17, 3PM

Newt's Meth

finds x where $f(x) = 0$.

"estimate $\sqrt{17}$"

build $f(x)$ with $f(\sqrt{17}) = 0$

$f(x) = x^2 - 17$

$\sqrt[3]{30} \rightarrow f(x) = x^3 - 30$

Try $\int_0^3 2x + x^2 \, dx$

a) FTC: $x^2 + \frac{1}{3}x^3 \Big|_0^3 = 3^2 + \frac{1}{3} \cdot 3^3 - \left(0^2 + \frac{1}{3} \cdot 0^3\right)$

$= 9 + 9 = 18$

b) Riemann sum:

$$\Delta x = \frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n}$$

$$x_i = a + i\,\Delta x = i \cdot \frac{3}{n} = \frac{3i}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \overbrace{\left(2\left(\frac{3i}{n}\right) + \left(\frac{3i}{n}\right)^2\right)}^{f(x_i)} \cdot \overbrace{\frac{3}{n}}^{\Delta x}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i}{n} + \frac{9i^2}{n^2}\right) \cdot \frac{3}{n}$$

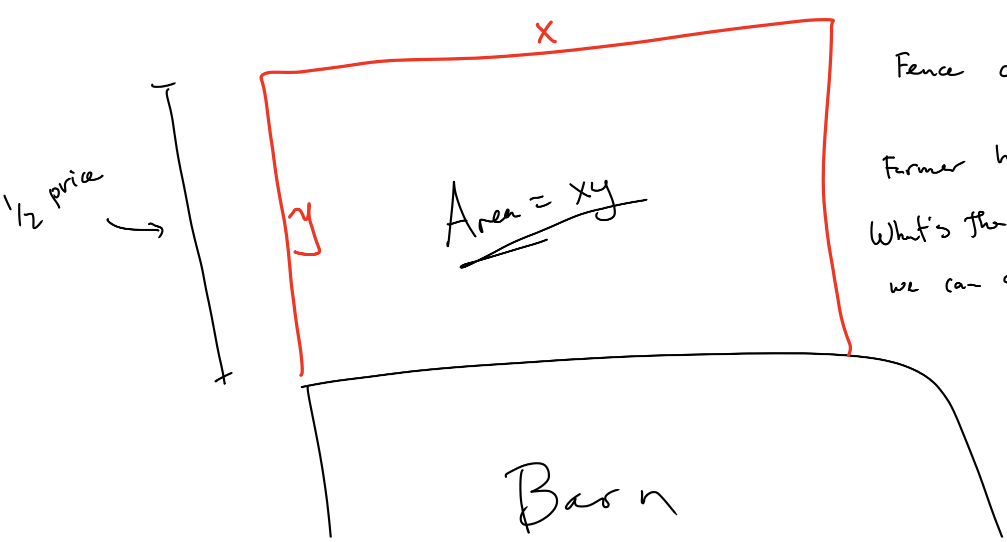
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{18i}{n^2} + \frac{27i^2}{n^3}$$

$$= \lim_{n\to\infty} \frac{18}{n^2} \sum_{i=1}^{n} i + \frac{27}{n^3} \sum_{i=1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{18}{n^2} \frac{n(n+1)}{2} + \frac{27}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{18}{2} + \frac{27 \cdot 2}{6}$$

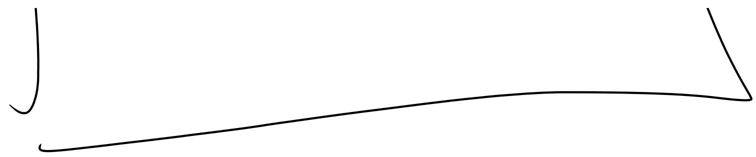
$$= 9 + 9 = 18$$

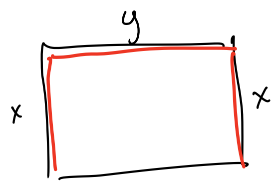


Fence costs \$20/ft,

Farmer has \$5000.

What's the max area we can enclose?





total cost $= 5000$

$$10x + 20y + 20x = 5000$$

$$30x + 20y = 5000$$

$$20y = 5000 - 30x$$

$$\boxed{y = 250 - \tfrac{3}{2}x}$$

$$A = xy = x\left(250 - \tfrac{3}{2}x\right)$$

$$\boxed{A(x) = 250x - \tfrac{3}{2}x^2}$$

interval: smallest x: $x = 0$.

biggest x is when $y = 0$: $30x = 5000$

$x = 500/3$.

$A(x)$ is zero for both of theses.

crit #s $A'(x) = 250 - 3x$

$A' = 0$: $250 - 3x = 0$

$3x = 250$

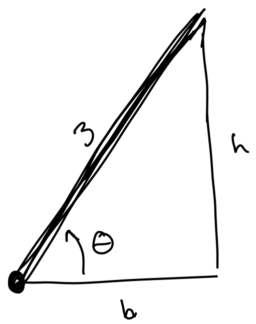
$x = 83.\bar{3}$

The max area is achieved when $x = 83.\bar{3}$

This Area is:

$$250x - \frac{2}{3}x^2$$

$$= \underline{250 \cdot 83.3 - \frac{2}{3} \cdot 83.3^2}$$



hyp is always length 3,

θ increases at 2 radians/sec.

How fast is the vertical side changing when $\theta = 45°$?

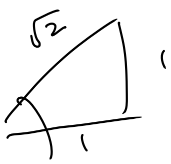
quantities: θ, h.

$$\sin\theta = \frac{h}{3} = \frac{1}{3}h$$

solve

imp diff: $$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{3} \cdot \frac{dh}{dt}$$

$\theta = \pi/4$ $\qquad$ $\frac{d\theta}{dt} = 2$



$$\cos\pi/4 \cdot 2 = \frac{1}{3}\frac{dh}{dt}$$

$$\frac{1}{\sqrt{2}} \cdot 2 = \frac{1}{3}\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{6}{\sqrt{2}}$$