Math 1015: Homework #5 selected answers

Question 1. Consider this weighted voting system: $[18 : 6, 4, 4, 3, 2, 1]$. In each part, briefly explain (don't just say "yes" or "no"):

a) Are there any dictators?

b) Are there any voters with veto power?

c) Are there any dummies?

Answer.   a) There are no dictators, since all weights are less than 18.

b) The 6 has veto power, since the 6 is necessary in order to reach 18. $(4 + 4 + 3 + 2 + 1 = 14$, which is not enough.) The 4's also have veto power, since $6 + 4 + 3 + 2 + 1 = 16$ which is not enough. The 3 also has veto power since $6 + 4 + 4 + 2 + 1 = 17$ is not enough. The 2 and 1 do not have veto power since you can get to 18 without each one of them.

c) None of them are dummies. Even the 1 can be important sometimes, like in $6 + 4 + 4 + 3 + 1 = 18$.

Question 3. Look up the numbers for the weights used in the US Electoral College system. (Try Wikipedia "United States Electoral College", scroll down to section "Current electoral vote distribution".)
    In each part, briefly explain (don't just say "yes" or "no"):

a) What is the quota in this system?

b) Is any state a dictator in this system?

c) Does any state have veto power?

d) Is any state a dummy?

Answer.   a) The quota is 270: this is how much you need to win.

b) There is no dictator, since no state has 270 by itself.

c) There is no state with veto power, since no individual state is necessary in order to get to 270.

d) There is no dummy state: even the smallest states can in theory make a difference in some situations.

Question 5. Let's say we have 4 people voting, and their weights are 8, 5, 4, and 2. What are the allowable values for the quota, according to the inequalities we discussed in class?

Answer. From class, we have the inequalities:

$$\frac{1}{2}(w_1 + \cdots + w_n) < q \leq w_1 + \ldots w_n.$$

The sum of the weights is $8 + 5 + 4 + 2 = 19$, so the above means that q must be greater than 9.5 (ie 10 or more), and less than or equal to 19.

Question 6. Compute the Shapley-Shubik power index for $[15 : 10, 7, 3]$.

Answer.

perms	weights	pivotal
ABC	10 7 3	B
ACB	10 3 7	B
BAC	7 10 3	A
BCA	7 3 10	A
CAB	3 10 7	B
CBA	3 7 10	A

So the SS index is A: 3/6, B: 3/6, C: 0/6