	Majority	Unanimity	CWC	Mono	IIA
Plurality	✓	✓	✗	✓	✗
RCV	✓	✓	✗	✗	✗
Condorcet	✓	✓	✓	✓	✓
Borda	✗	✓	✗	✓	✗
Dictatorship	✗	✓	✗	✓	✓

Independence From Irrelevant Alternatives IIA

making "irrelevant" choices on a ballot (shuffling around the losers) won't change the result.

Ex

	3	2
2	A	$\boxed{B}$
1	$\boxed{B}$	C
0	C	A

using Borda:

A: $3 \cdot 2 + 2 \cdot 0 = 6$

B: $3 \cdot 1 + 2 \cdot 2 = 7$

C: $3 \cdot 0 + 2 \cdot 1 = 2$

B wins.

Let's chg B C A to B A C. This only effects the losers.

Now it's:

	3	2
2	A	B
1	B	A
0	C	C

A: $3 \cdot 2 + 2 \cdot 1 = 8$

B: $3 \cdot 1 + 2 \cdot 2 = 7$

C: $3 \cdot 0 + 2 \cdot 0 = 0$

A wins!

IIA If there is a winner & we change ballots without adjusting anyone's ranking relative to the winner, then the results won't change.

irrelevant changes look like:

winner $\to \begin{array}{c}\boxed{B}\\ A\\ C\end{array} \to \begin{array}{c}\boxed{B}\\ C\\ A\end{array}$ or winner $\to \begin{array}{c}A\\ B\\ \boxed{C}\end{array} \to \begin{array}{c}B\\ A\\ \boxed{C}\end{array}$

$\begin{array}{c}A\\ \boxed{B}\\ C\end{array} \to \begin{array}{c}C\\ \boxed{B}\\ A\end{array}$ Doesn't count as an irrelevant change

On HW: show plurality & RCV also don't satisfy IIA.

Condorcet's Method does satisfy IIA:

Imagine X is the winner using Condorcet's method, then we change some ballots without changing any rankings relative to X.

∴ Then any pairwise comparison X vs Y will be unaffected.

⋮

Therefore X is still the winner using Condorcet's method.

Is there a practical system that satisfies all these criteria?

NO!

1950s <u>Arrow's Impossibility Theorem</u> The only system which always chooses a winner and satisfies unanimity & IIA is dictatorship!

Arrow proved no such method can exist logically.

"you can't prove a negative" yes you can!

Takeaway: No <u>perfect</u> voting system can exist.

Manipulability

With more sophisticated systems, people wonder about gaming the system.

Voting in some way other than my true preferences in order to get a better outcome.

Def A voting system is manipulable if: voters can achieve a better result (in their own opinion) by voting differently from their true preferences.

This is bad: If the system is manipulable, then the voters must decide their prefences, but also choose a good strategy.

Plurality is manipulable:

Bush - Gore - Nader

2.9	2.9	.1
B	G	N
G	B	G
N	N	B

using plurality, B wins

N / G / B could vote instead for G / N / B,

then G wins, which is a better outcome in their opinion.

RCV is also manipulable:

6	4	3
A	C	B
B	B	C
C	A	A

RCV: Round 1 eliminate B

Round 2 C wins.

If one A / B / C changes to B / A / C:

5	1	4	3
A	B	C	B
B	A	B	C
C	C	A	A

RCV: Round 1: eliminate C & B.

A wins!

(this is preferable to the A / B / C voters)

The Gibbard - Satterthwaite Theorem

All systems are manipulable except dictatorship.

Borda is regarded as super bad regarding manipulations.

BGN: using Borda, assume all voters vote strategically.

Strategy: Put my favorite candidate on top, put the biggest threat opposition on bottom.

BGN: True votes:

2.9	2.9	.1
B	G	N
G	B	G
N	N	B.

Strategically, they would vote:

	2.9	2.9	.1
2	B	G	N
1	N	N	G
0	G	B	B

Borda:

B: $2.9 \cdot 2 + 2.9 \cdot 0 + .1 \cdot 0 = 5.8$

G: $2.9 \cdot 0 + 2.9 \cdot 2 + .1 \cdot 1 = 5.9$

N: $2.9 \cdot 1 + 2.9 \cdot 1 + .1 \cdot 2 = 6.0$

N wins!

Voting strategically with Borda can lead to absurd results.