

Weighted Voting

[20 : 12, 10, 5, 1, 1]

↑
quota

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weights of the voters

Who has the power? & how much?

The 12 & 10 have equal power,

the 5, 1, 1 each have no power.

12	has	50%.
10	has	50%.
5		
1	have	0%.
1		

[30 : 10, 10, 10, 9]

10s have equal power,

9 has no power.

Which weighted scenarios are worth discussing?

$$[20 : 1, 1, 1]$$

The weights are too small to reach the quota.

$$\text{in } [q : w_1, w_2, \dots, w_N]$$

For it to make sense,

$$q \leq w_1 + w_2 + \dots + w_N$$

$$\leq \leftarrow \text{less than or equal.}$$

Another issue:

$$[2 : 5, 5, 4, 1]$$

Here, we can pass the law if the 4 votes
for it, even if the
5, 5, 1 are against.

If the quota is too low, it can pass
even if a majority are opposed.

So the quota must be at least 50 %.

of the total weights.

$$q > \frac{1}{2}(w_1 + w_2 + \dots + w_n)$$

So

$$\frac{1}{2}(w_1 + \dots + w_N) < q \leq w_1 + \dots + w_N$$

Basic measures of power:

$$[10: 12, 4, 2, 3]$$

↑

12 meets the quota all by themselves,
which means the others have no power.

We say the 12 is a dictator.

(any # bigger than the quota)

$$[30: 10, 10, 10, 9]$$

no 10 is a dictator

The 9 has no power at all

When a voter has no power, they are called a dummy

If someone is a dictator, all others are dummies.

Ex Is there any dictator or dummies here?

$[10: 5, 4, 1, 1]$

No dictator. (no voter meets the quota)

No dummies: even the 1s can contribute
as in $5 + 4 + 1$.

$[12: 9, 5, 4, 2]$

No dictator.

Dummies: 2 is a dummy

4 is not: $9 + 4 = 13$, 4 is important.

5 & 9 are not dummies

Here, the 9 can't make the quota on their own,
but the 9 is necessary to reach the
quota.

A "yes" from \mathcal{P} doesn't force a "yes" result,
but a "no" . . . does force a "no" result.

We say \mathcal{P} has veto power

Def If some voter is necessary to meet
the quota, we say they have veto power

In $[30: 10, 10, 10, \mathcal{P}]$

No dictator, \mathcal{P} is a dummy,
each of the 10s has veto power.

In $[30: 10, 10, 10, 10, \mathcal{P}]$

No dictator, \mathcal{P} is a dummy,
no veto power.

[10: 14, 3, 1]

Dict: 14
Dum: 3, 1
Veto: 14

[10: 7, 3, 3, 2]

Dict: None
Dum: 2
Veto: 7 (not 3)

[10: 5, 4, 3, 2]

Dict: None
dummy: none
Veto: 5

[10: 7, 3, 2]

Dict: none
dum: 2
Veto: 7 at 3.

Specific measures of % of power.

2 ways to do it that we'll discuss,
different methods giving different answers.

Shapley - Shubik Power Index (SS)

Imagine the voters cast their votes in some order, the total will increase until it reaches the quota (or not)

SS measures power by how often a voter is "the pivotal vote"

Each reordering is called a voting permutation

Def The Shapley - Shubik power index of some voter is the fraction of all permutations in which they are pivotal.

List all permutations, decide who is pivotal each time, make fractions.

Ex [16 : 12, 10, 5] Find the SS.

<u>perms</u>	<u>weights</u>	<u>pivotal</u>	
A B C	12, 10, 5	B	A: $\frac{4}{6} = 66.6\%$ B: $\frac{1}{6} = 16.6\%$ C: $\frac{1}{6} = 16.6\%$
A C B	12, 5, 10	C	
B A C	10, 12, 5	A	
B C A	10, 5, 12	A	
C A B	5, 12, 10	A	
C B A	5, 10, 12	A	