

# Shapley - Shubik

$$[12 : 8, 7, 1]$$

A    B    C

perms	weights	pivotal	
A B C	8, <u>7</u> , 1	B	A: $3/6 = 50\%$
A C B	8, 1, <u>7</u>	B	B: $3/6 = 50\%$
B A C	7, <u>8</u> , 1	A	C: $0/6 = 0\%$
B C A	7, 1, <u>8</u>	A	
C A B	1, 7, <u>8</u>	B	
C B A	1, 7, <u>8</u>	A	

Try

$$[12 : 7, 6, 5]$$

perms	weights	pivotal	
A B C	7, <u>6</u> , 5	B	A: $4/6$
A C B	7, <u>5</u> , 6	C	B: $1/6$
B A C	6, <u>7</u> , 5	A	C: $1/6$
B C A		A	
C A B		A	
C B A		A	

$$7 = \frac{4}{6}$$

$$6 = \frac{1}{6}$$

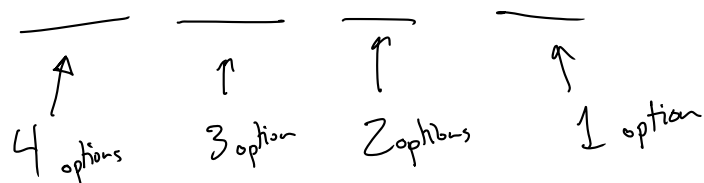
$$5 = \frac{1}{6}$$

With 3 voters, there are 6 perms.

With 4, how many?

A, B, C, D

Each perm. is about filling in:



Basic fact If I make two independent choices with  $x$  ways to choose the first, and  $y$  different ways to choose the second, then the total # of ways to choose is  $x \cdot y$ .

So for perms of 4 things



There are  $4 \cdot 3 \cdot 2 \cdot 1 = 24$  possibilities.

For perms of  $N$  things, there  
are  $\underbrace{1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot N}$  permutations.

written as  $N!$

"N factorial"

$$\text{So } 4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$$

$$3! = 1 \cdot 2 \cdot 3 = 6$$

$$5! = 120$$

$N!$  is super big!

Favorite Math facts:

$\sim 1700$

Stirling's Formula For large  $N$ ,

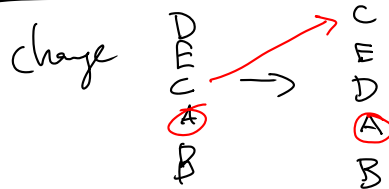
$$N! \approx \sqrt{2\pi N} \cdot \frac{N^N}{e^N}$$

## Facts about SS:

- IF  $X$  is a dummy, then  $X$  is never pivotal, so their SS fraction is  $\frac{0}{1} = 0\%$
- IF  $X$  is a dictator, then  $X$  is always the pivotal voter, so their SS is 100%
- IF  $X$  has veto power, harder to say, but it turns out the SS will be at least  $\frac{1}{N}$

<u>4</u>	<u>3</u>	<u>3</u>	<u>2</u>
A	B	C	D
B	A	A	E
C	C	B	C
D	D	D	A
E	E	E	B

Plurality does not satisfy IIA



IIA IF  $X$  is the winner, and we make a change which doesn't move

anyone past the winner, then  
X should still win.

Rand. Dict is good for repeated  
low-stakes decisions.

$\frac{3}{A}$	$\frac{2}{C}$	$\frac{2}{B}$	$\frac{2}{C}$
<del>B</del>	<del>B</del>	A	B
C	A	C	A

A wins with prob:  $\frac{3}{9}$

B . . . . . :  $\frac{2}{9}$

C . . . . .  $\frac{4}{9}$

[10 : A B C  
6, 5, 4]

A B C	6, 5, 4
A C B	6, 4, 5
B A C	5, 6, 4
B C A	5, 4, 6
C A B	4, 6, 5
C B A	4, 5, 6

No dict.  
No dummy  
A:  $\frac{4}{6}$  G has veto power  
B:  $\frac{1}{6}$   
C:  $\frac{1}{6}$