

Banzhaf Power Index

Banzhaf was a lawyer in 1960s
in Nassau County - 6 districts

	<u>weight</u>	
Hempstead #1	31	
" #2	31	
N. Hempstead	28	need 58 votes to win.
Oyster Bay	21	
Glen Cove	2	
Long Beach	2	

B. realized the 21, 2, and 2 are all dummies.

this is 22% of the county who have
no voting power at all.

B created his power index to "prove" it
was unfair.

In 1993, NY banned weighted voting

Same idea published in 1946 - Penrose

1971 - Coleman

Banzhaf - 1965

Consider all possible combinations of voters
(not in a particular order)
who can meet the quota.

A B C
[16: 12, 10, 5]

<u>combos</u>	<u>total weight</u>
A B C	27
A B	22
A C	17

In each combo, we
identify critical voters

A B C	15
A B	12
A C	10
B C	5
A	0
B	0
C	0

A voter is critical in some combo
when their weight is necessary to reach
the quota.

[16: 12, 10, 5]

<u>combos</u>	<u>total weight</u>
A B C	27
A B	22
A C	17

Who is critical in this combo?
 A is required (if we omit A, weight drops to 15, too small)

A is critical,
 B is not critical
 C is not critical.

in AB : 22, A is critical, B is critical

in AC : 17, A & C are both critical.

<u>combos</u>	<u>total weight</u>	<u>critical</u>		
		<u>A</u>	<u>B</u>	<u>C</u>
A B C	27	✓		
A B	22	✓	✓	
A C	17	✓		✓

Final answer: A: $3/5 = 60\%$

B: $1/5 = 20\%$

C: $1/5 = 20\%$

$$A = \frac{\# \text{ of times A is critical}}{\text{total \# of criticals}}$$

SS for this one was

A: $4/6 = 66.6\%$

B: $1/6 = 16.6\%$

C: $1/6 = 16.6\%$

2016 US President

Trump won, his combo added to 306
which states are critical in this combo?
in Trump's combo, only TX is critical.

2020

Biden's combo was the winner,
and only CA was critical.

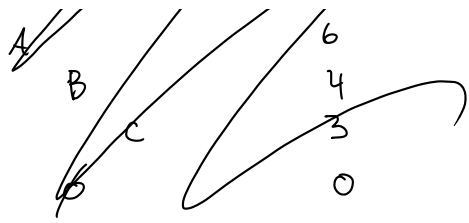
2000

Bush's combo won, all states were
critical. (if any had gone to Gore,
B would not meet the quota)

Find the Banzhaf for

$[10: 6, 4, 3]$

Combos	total weights	critical		
		A	B	C
A B C	13	✓	✓	
A B	10	✓	✓	
<hr/>				
A				✓
A B C				✓



$$A: 2/4 = 50\%$$

$$B: 2/4 = 50\%$$

With N voters, SS uses $N!$ perms.

How many combos will there be?

with A, B, C, there's 8.

How many combos with A, B, C, D?

like:

A B D

$\overline{2}$ $\overline{2}$ $\overline{2}$ $\overline{2}$

each is either the letter, or blank.

So the total # of combos is

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16.$$

For N voters, we will have 2^N combos.

Try Banzhaf for

A B C D

[8: 5, 4, 3, 1]

critical

	Compos	total	A	B	C	D
using 3	A B C D	13				
	A B C	12	✓			
	A B D	10	✓	✓		
	A C D	9	✓		✓	
	B C D	8		✓	✓	✓
using 2	A B	9	✓	✓		
	A C	8	✓		✓	
	A D	6				✓
	B C	7			✓	✓
	B D	5				✓
using 1	C D	4			✓	✓
	A		✓			
using 0	B			✓		
	C				✓	
	D					✓
	∅					

A: 5/12 B: 3/12 C: 3/12
D: 1/12

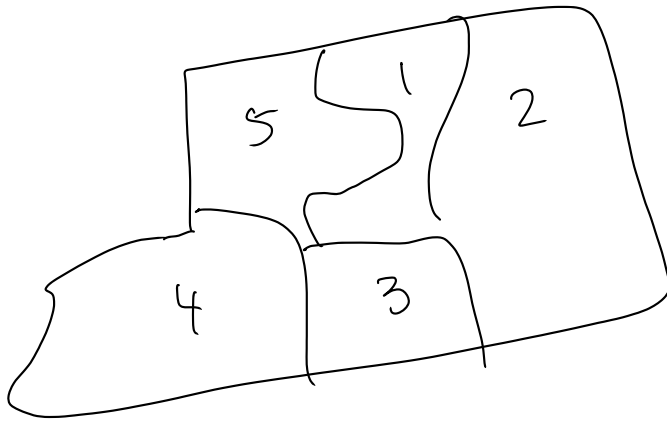
IF X is a dictator, then their Banzhaf score will be 100%

IF X is a dummy, they are never critical, so their Banzhaf is 0%.

Gerrymandering

About the arrangements of districts when voting.

CT has 5 US Congress districts.



in CT as a whole D/R split is
about 65% / 35%

our US Congress reps should go

3/2 or 4/1
D/R D/R

In CT, it's 5/0 all D's.