Convex Hull Ratio

$$CH = \frac{\text{area of original}}{\text{area of conv. hull}}$$

CH is a # between 0 & 1.

a nice shape has $CH \approx 1$,
a weird shape has $CH \approx 0$.

Areas of weird shapes are hard to find.

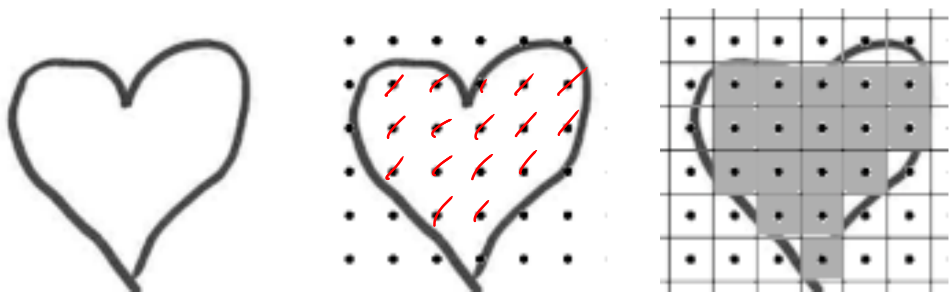
A tool for measuring areas:

The Dot Planimeter

"area measuring tool"

The fabulous dot planimeter

Finding the area of a weird shape is hard! So we superimpose dots on the picture, separated by 1cm (or whatever). Count the dots that lie inside the shape.

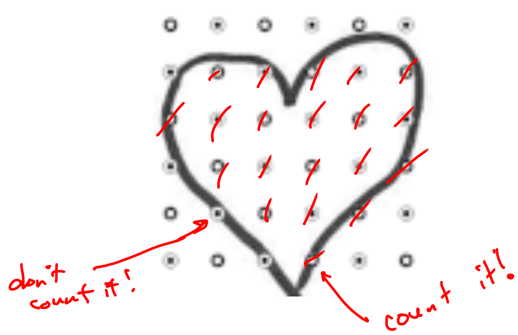


Counting the dots is like counting the squares, so gives a pretty good estimate of the area. Looks like 17 cm^2 (or whatever).

What if it's on the line?

A dot on the line represents an area of 1/2. So those should be counted as half. Or equivalently, only count up half of them.

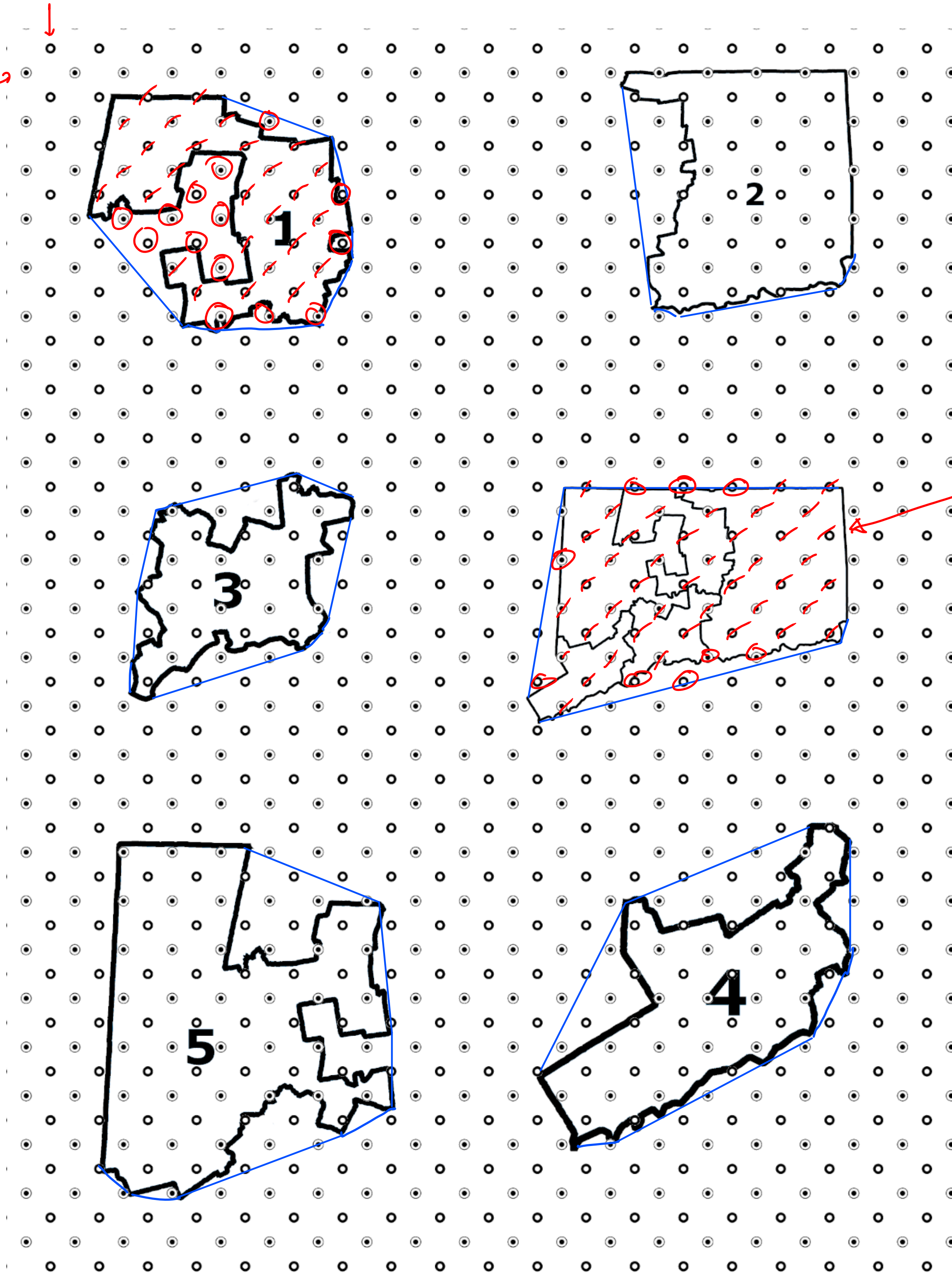
- If the dot isn't on the line, count it no matter what.
- If the dot is on the line and looks like ○, count it.
- If the dot is on the line and looks like ⊙, don't count it.



#1: $CH = \frac{\text{original area}}{\text{area of conv. hull}} = \frac{27}{41} = 66\%$

count this one if on the line

skip if on the line.



$\frac{42}{51} = 82\%$

My answers:

#1: 66%

#2: 87%

#3: 69%

#4: 66%

#5: 76%

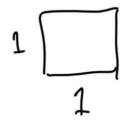
Isoperimetric Quotient (IQ)

IQ is a measure of area vs perimeter.

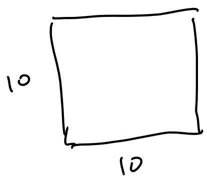
basically: $\frac{\text{area}}{\text{perimeter}}$.

$\frac{A}{P}$ A ← area, P ← perimeter

$\frac{A}{P}$ is OK, but doesn't scale properly:

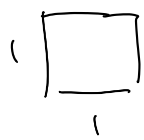


then $A = 1$, $P = 4$ so $\frac{A}{P} = \frac{1}{4}$

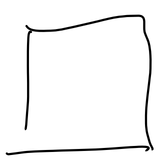


then $A = 100$, $P = 40$ $\frac{A}{P} = \frac{100}{40} = \frac{10}{4} = 2.5$

Turns out, we need to use $\frac{A}{P^2}$



$A = 1$, $P = 4$ $\frac{A}{P^2} = \frac{1}{16}$



$A = 100$, $P = 40$ $\frac{A}{P^2} = \frac{100}{1600} = \frac{1}{16}$

We'll use $\boxed{IQ = \frac{4\pi A}{P^2}}$

The 4π is there so that a circle has $IQ = 1$.

IQ in politics is called the Polsby-Popper score.

first used in a 1927 paper on paleontology

IQ is always between 0 & 1.

close to 1 is "good" - close to a circle

close to 0 is "bad" - like a worm.

US Congress districts average IQ is .23.

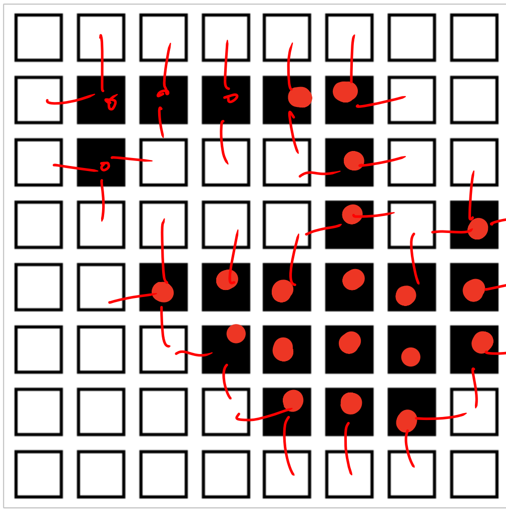
MD #2 has IQ = .02

Bad news: no good easy way to measure perimeter on paper

$A=23$

$P=36$

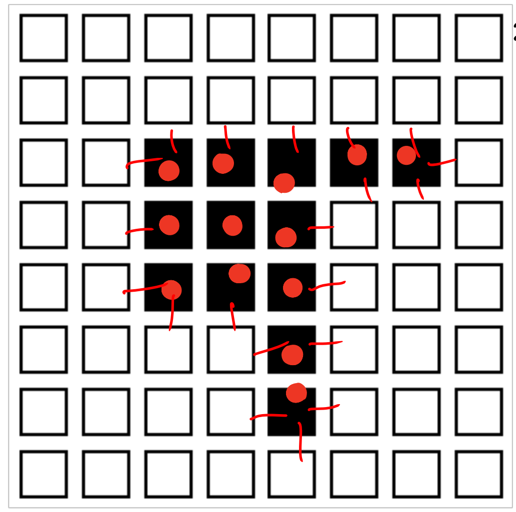
$IQ = \frac{4\pi \cdot 23}{36^2} = .22$



$A=13$

$P=20$

$\frac{4\pi \cdot 13}{20^2} = 40\%$



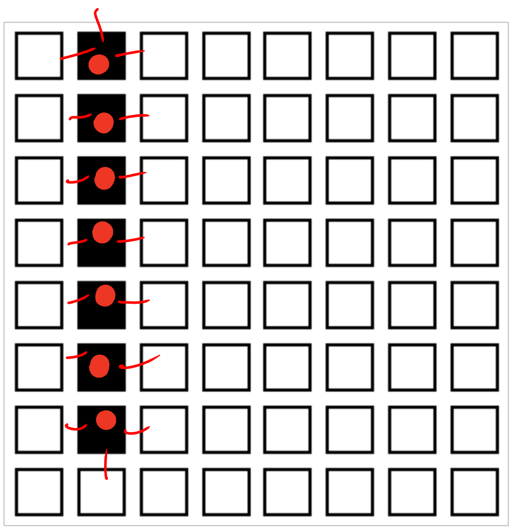
$A=7$

$P=16$

$\frac{4\pi \cdot 7}{16^2}$

"

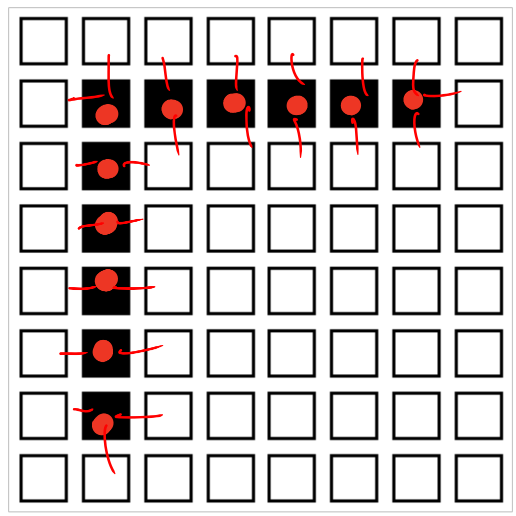
$.08$



$A=11$

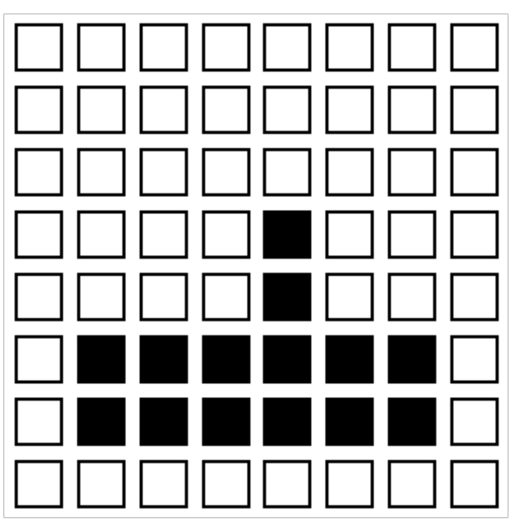
$P=24$

$\frac{4\pi \cdot 11}{24^2}$



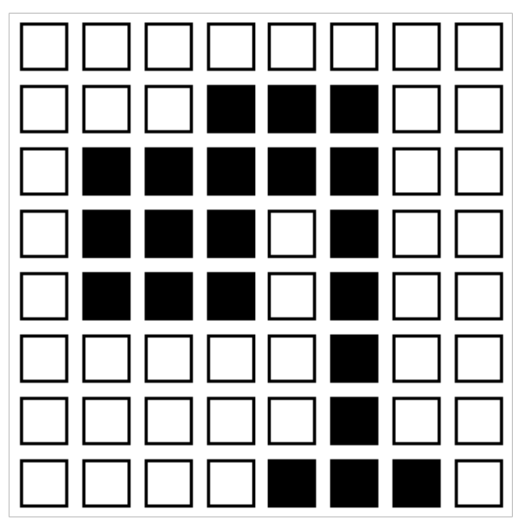
$A=14$

$P=20$



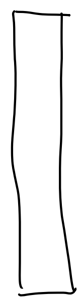
$A=21$

$P=32$



Area: just count the squares

IQ for



is quite small $\sim 8\%$

For this shape, $CH = \dfrac{\text{area of original}}{\text{area of hull}} = 1$

(these areas are equal)

A long skinny rectangle has great CH score, very bad IQ score.