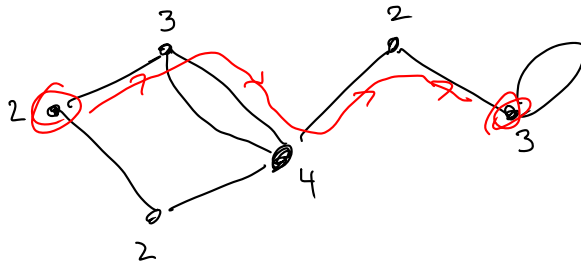


# Graph Theory

A graph is like:

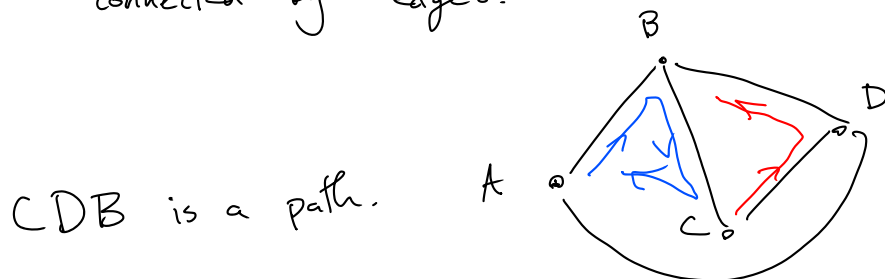


The dots are vertices (vertex)  
lines are edges

Terms The degree of a vertex is the # edges touching it. (looped edges count twice)

A looped edge is an edge from a vertex to itself

A path is a sequence of vertices connected by edges.

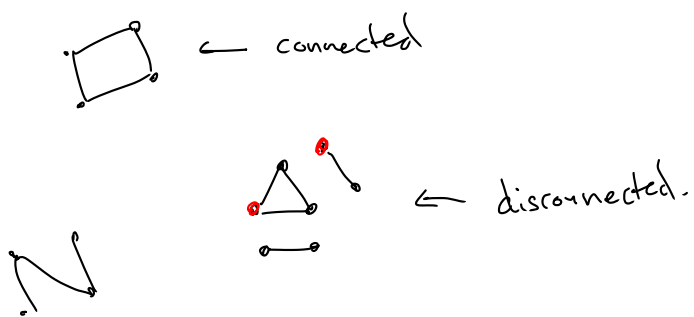


CDB is a path.

ABCA is not a path in this graph.  
(no edge from C to A)

A circuit is a path beginning & ending at the same vertex.  $CDBC$  is a circuit.

A graph is connected when any vertex can be reached by a path from any other vertex.



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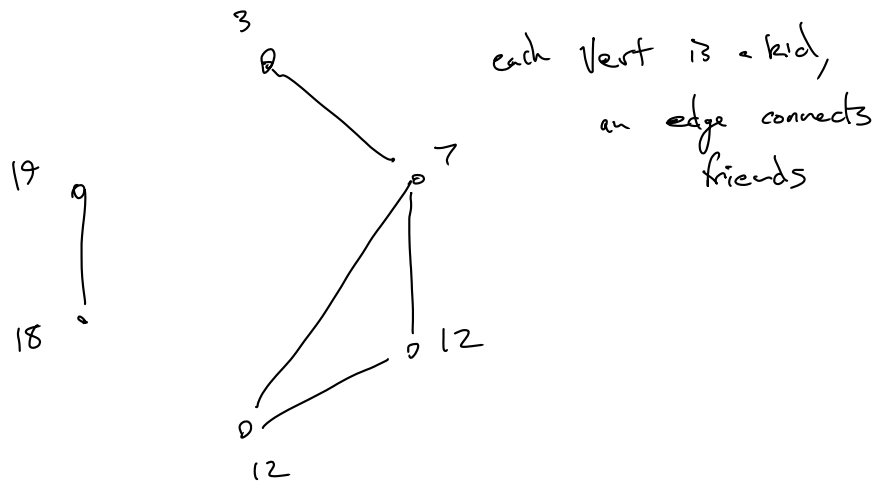
Imagine a graph...

Each vertex is one of my kids, their ages are

3, 7, 12, 12, 18, 19

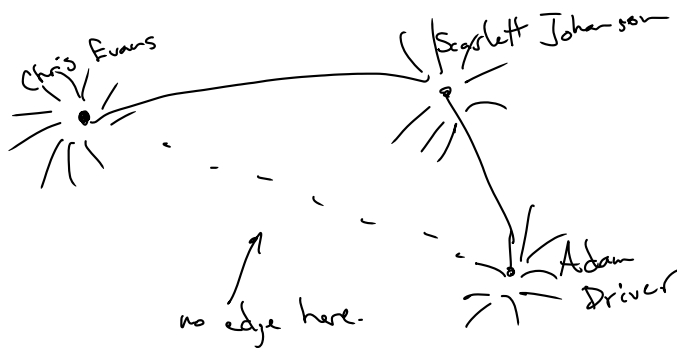
They are friends with ones who are within 5 years of age difference.

Represent it as a graph



Disconnected! Degree reps # of friends

Consider verts are Hollywood actors,  
an edge connects 2 who were in a  
movie together.



- What does the degree represent? # of other actors they worked with
- Who has bigger degree? Tom Hanks or Macaulay Culkin

Tom Hanks

- Is it connected? (probably)
  - Britney Spears was in 1 film: Crossroads.  
So Britney's degree is small -  
however many people were in Crossroads.
- 

A graph where each vert is a FU full-time student, not abroad.

An edge connects 2 people who are in a class together.

What is your degree in the graph (approx)

prob about 100 ← the total # of people who are in some with you.

Is the graph connected? probably yes.

Full students, an edge connects two students if they're from the same state

• Joe from MA vs Skye from CA.

Joe has a higher degree than Skye.

Is it connected? NO

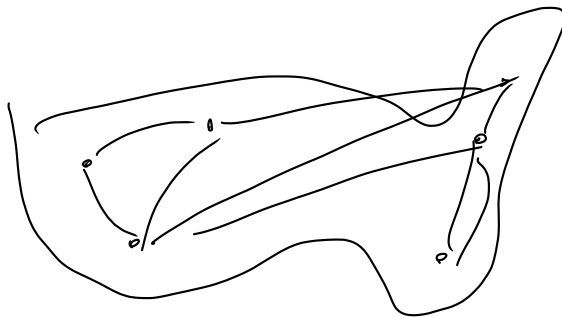
The graph has a different component for each state.

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Super-useful in the real world!

Road navigation Road networks look like graphs, the GPS lady is finding shortest paths from one point to another.

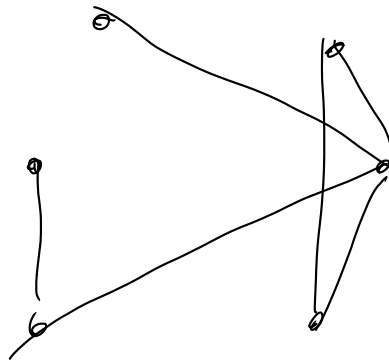
Airline Routes



Expressed as a graph  
High degrees represent airline "hubs"

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### Computer networks



Social Networks Each user is a vertex,  
relationships (following / friends / subscribers)  
are edges.

degree represents # of friends.