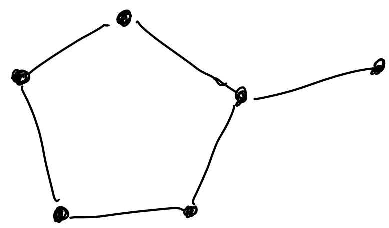
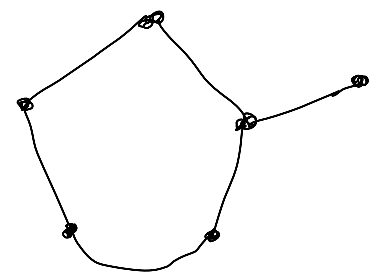
Graphs

What is a graph — really?

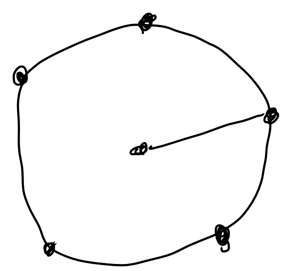


or



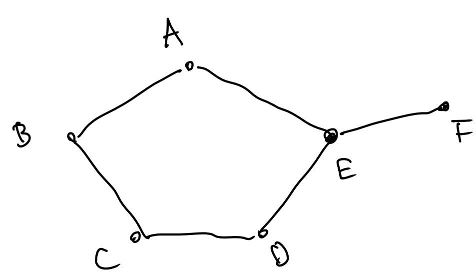
look slightly different, but we consider them to be the same.

Also same as:



The true definition of a graph:

A set of vertices, with a set of edges, described by pairs of verts.



This graph is: verts: $\{A, B, C, D, E, F\}$

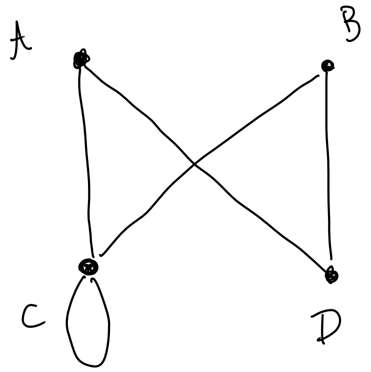
edges: $\{(A,B), (B,C), (C,D), (D,E), (A,E), (E,F)\}$

We say this graph is

$$\Big(\underbrace{\{A,B,C,D,E,F\}}_{\text{set of verts}}, \underbrace{\{(A,B),(B,C),(C,D),(D,E),(A,E),(E,F)\}}_{\text{set of edges.}}\Big)$$

Ex Draw this graph:

$$(\{A,B,C,D\}, \{(A,C),(B,C),(A,D),(C,C),(B,D)\})$$



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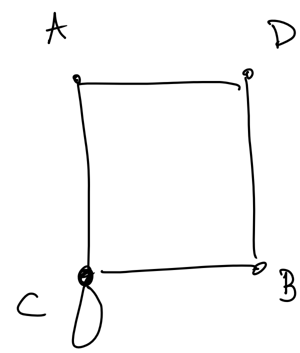
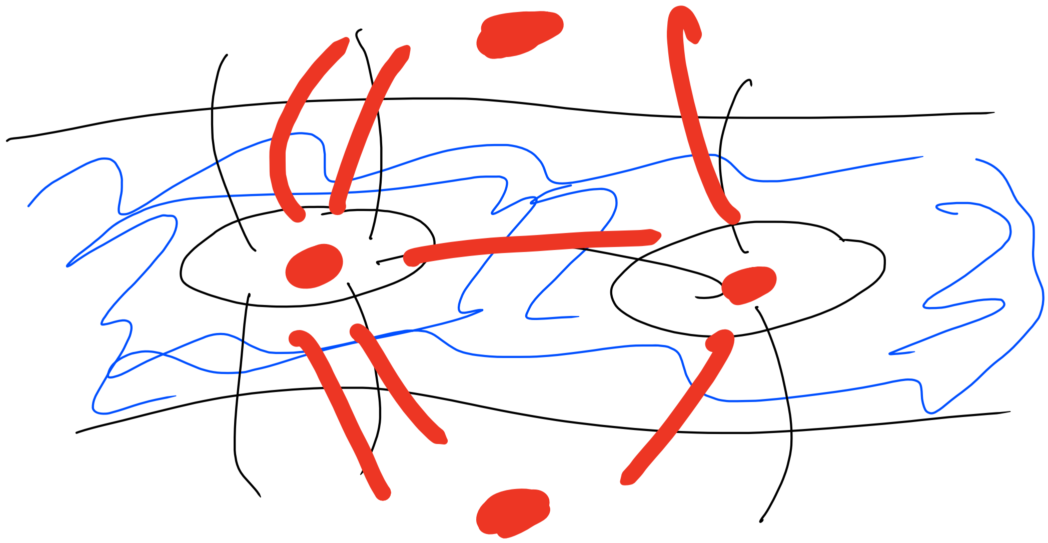


$$\underbrace{\text{Euler Circuits}}_{\text{Oiler}}$$

Original graph theory problem:

"7 Bridges of Königsberg"

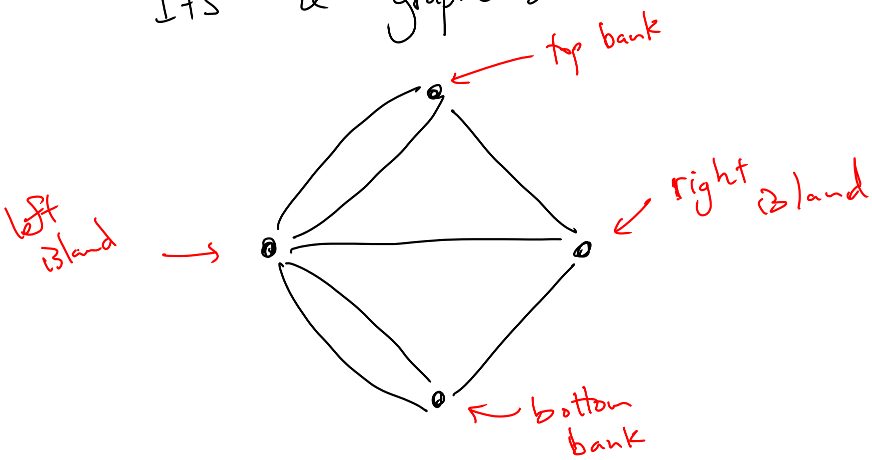


Can you walk across all 7 bridges without repeating any, and ending where you started?

It's impossible — but how do we think of this mathematically?

Looks like geometry, but really it's not.

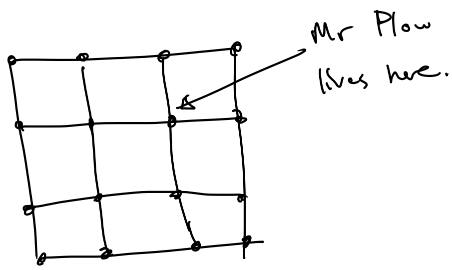
It's a graph!



Can we find a circuit which uses every edge once, no repetitions.

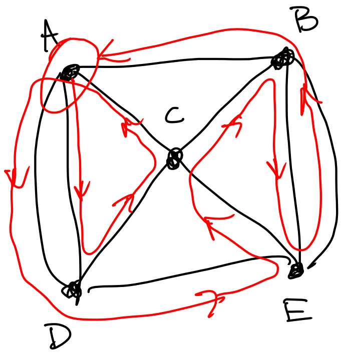
Def A circuit using each edge exactly once is called an <u>Euler circuit</u>.

Ex



Can Mr Plow drive every street once and end up back at home with no repeats?

If not, how many repeats are necessary?

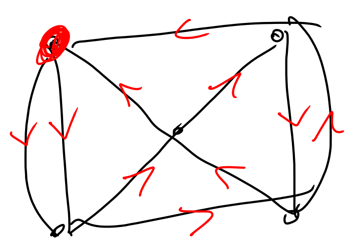


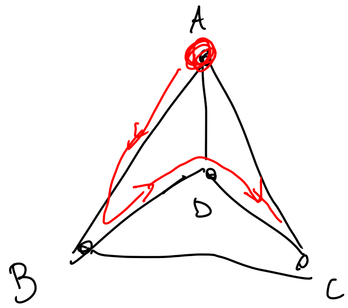
This one has an E. circ.

ADCADECBEBA

follow next to edges & verts

don't Do:





This does not have an E. circ.

Hard to say why not.

Language from Euler's Theorem:

"if and only if" or "iff"

" A if and only if B "

means that A & B are logically equivalent.

A & B mean exactly the same thing

OR: it means both: "if A, then B"

and "if B, then A"

Are these correct uses?

I'm dressed today iff I'm wearing pants

"dressed" is not equivalent to "pants"

I'm old enough to drink iff I'm 35 years old.

I've seen the Statue of Liberty ~~iff~~ I've been to NYC

x is an even number iff x is divisible by 2.

x is an even number iff x+1 is odd

Euler's Theorem For any connected graph G:

G has an Euler circuit iff all verts have even degree

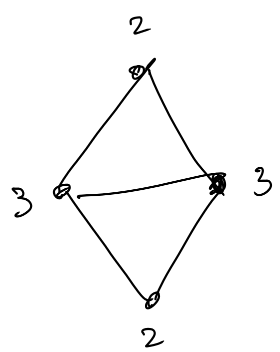
Deep & powerful! Says

Hard

Euler circs are equivalent to even degrees.

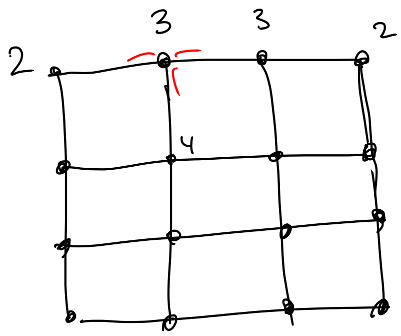
easy

Ex

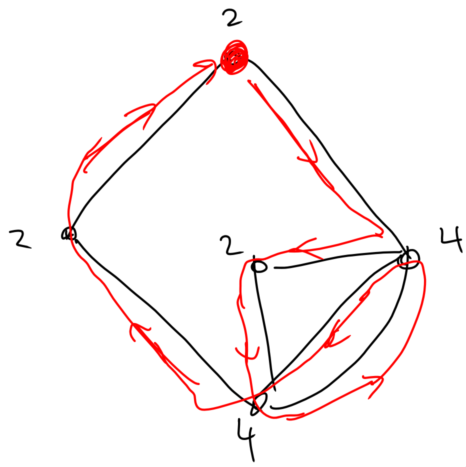


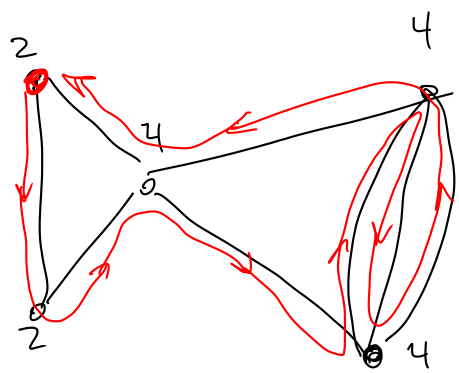
Not all even, so

no Euler circuit.

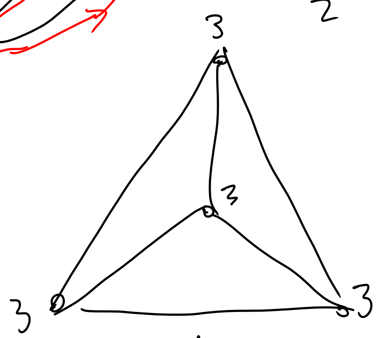


Not all even, so no Euler circuit.

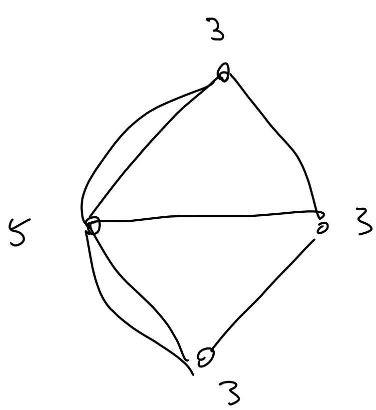




Either draw an Euler circ, or say it doesn't exist.



no Euler circ.

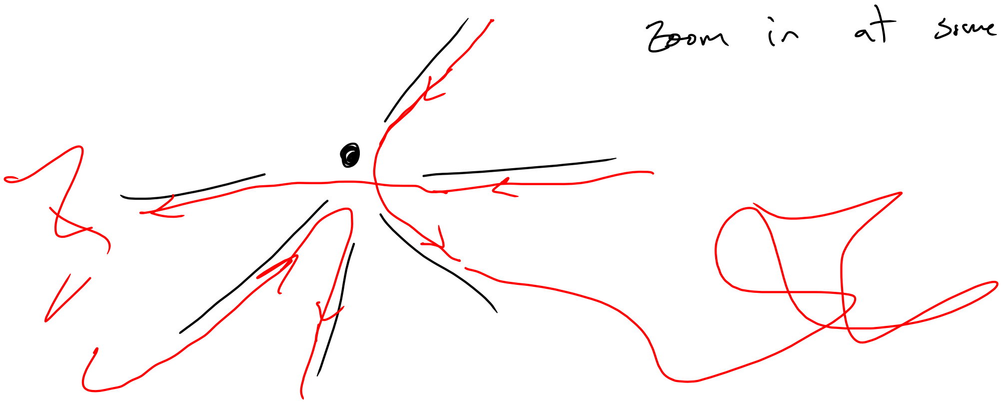


No Euler circuit.

Why is degree even equiv. to Euler circs?

Imagine there is an Euler circuit:

Zoom in at some vertex



Each vert must have some "arriving" edges, and the same # of "leaving" edges.

So the degree must be even.