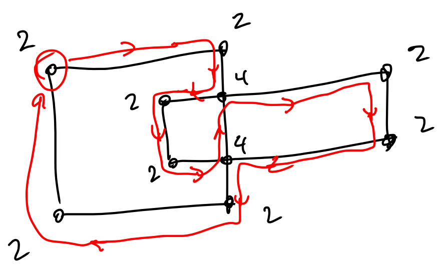
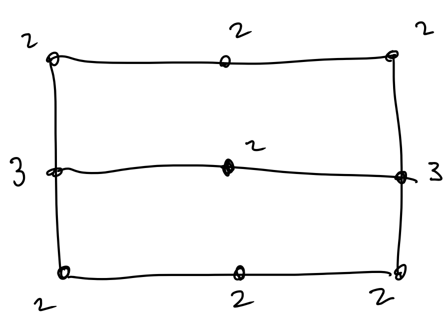
Euler Circuits

A circuit using each edge exactly once.

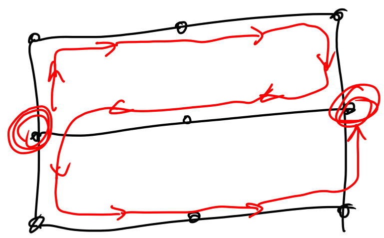


Euler's Theorem An Euler circuit exists if and only if all verts have even degree.

Related: Euler Path A path (not a circuit), which uses every edge exactly once.



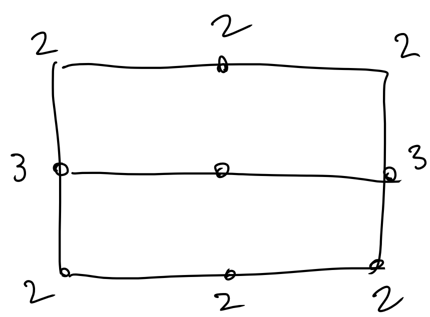
There are odds, so there's no Euler circuit.



This is an Euler path.

An E.P. looks like an E.C., except
the start pt has an extra "leaving" edge
end pt . . . " " ", "arriving" edge.

So all verts will be even degree except
start & end, which are odd.



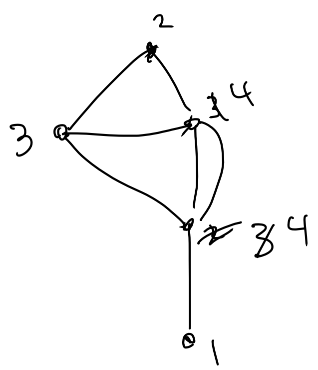
2 odds must be the start & end.

Thm There is an Euler circuit iff all verts have even degree,

There is an Euler path iff all verts have even degree except for 2 odds.

(if more than 2 odds, there is no E. circ or path)

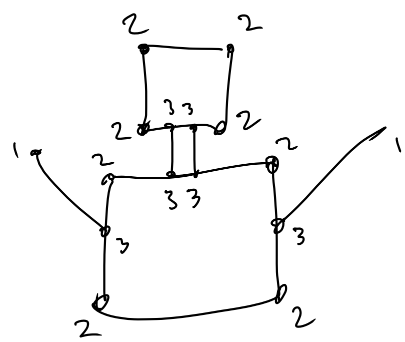
What if there's only 1 odd degree vert?



This is impossible

Thm Euler's sum-of-degrees theorem

In any graph, the sum of all degrees is even.



sum of all degrees is 36.

This is why no graph can have 1 odd, the rest even.

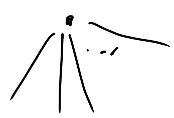
NFL True Stories:

At one time NFL had 2 conferences,
13 teams in each.

Guidelines: each team will play 14 games,
11 in-conference, 3 outside conference

This is impossible!

Imagine a graph of the 13 teams, & 11 in-conf. games.

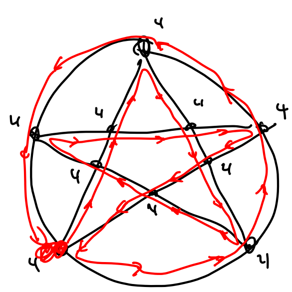


Each vert has degree 11.
13 verts.

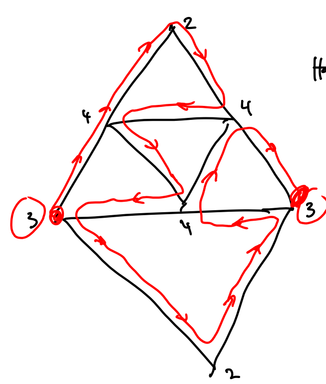
So total sum of degrees is

$11 \times 13 = 143$ which is odd!

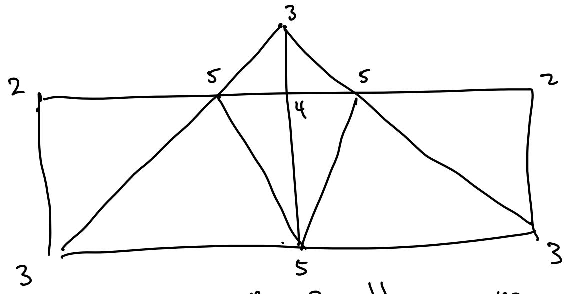
So this graph cannot exist.



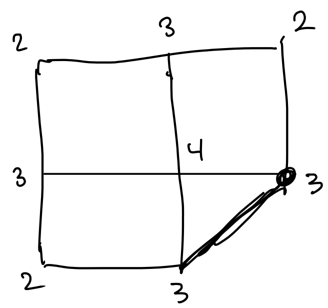
Has an E. circuit.



Has an E. path



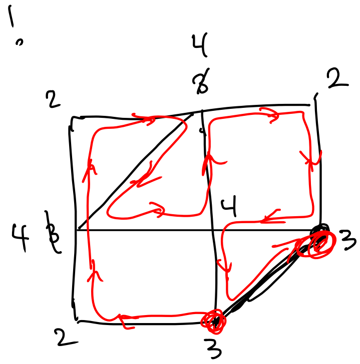
More than 2 odds, so no Euler circ or path.



Can Mr Plow drive each street without repeats?

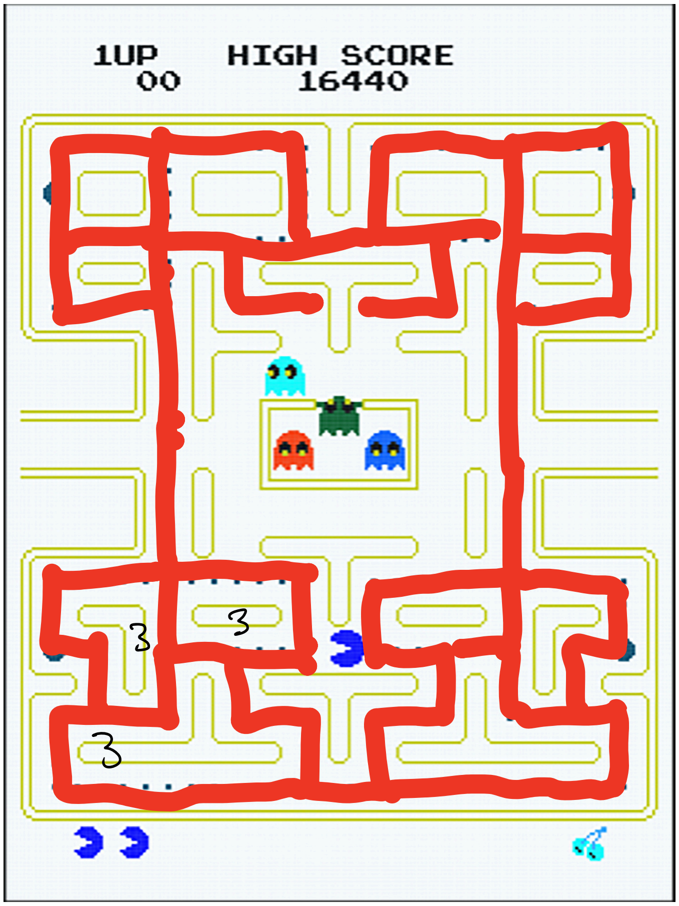
No! More than 2 odds, so we must repeat some.

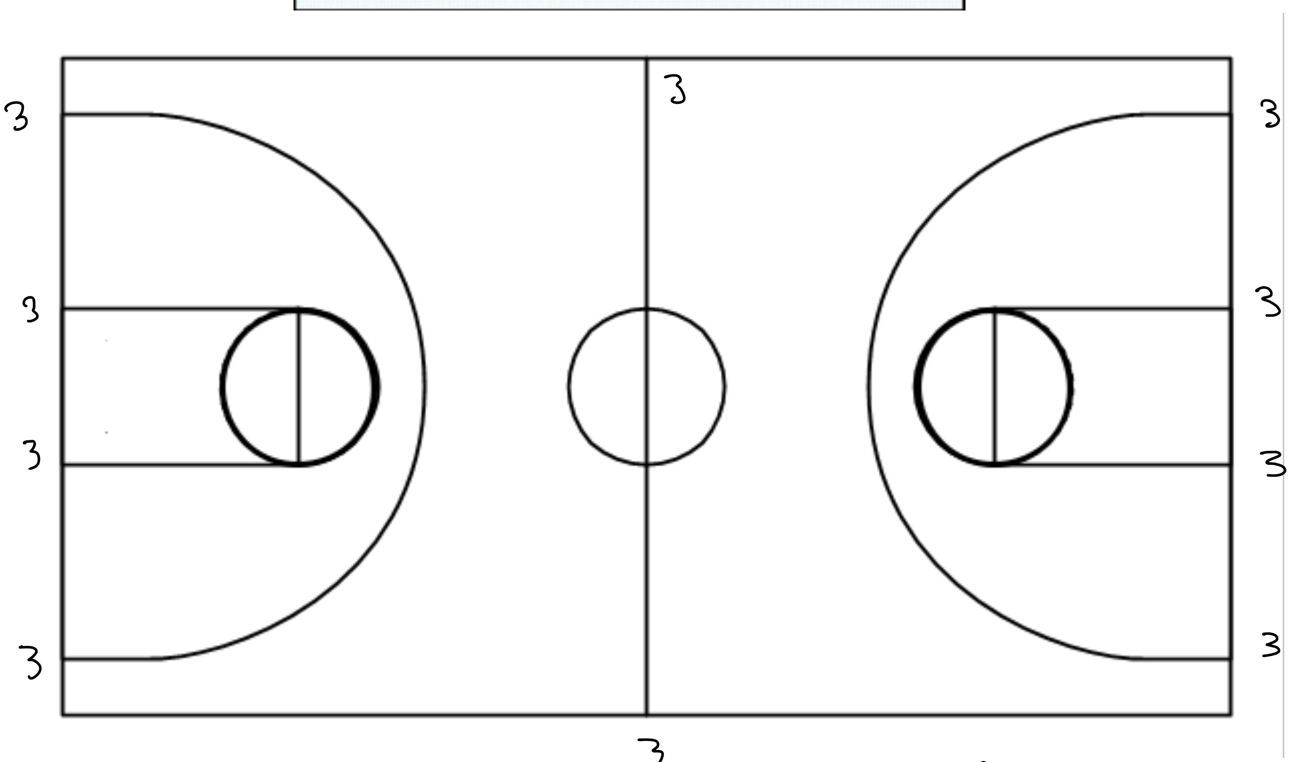
Ms Road wants to build a new road to help him!



Now it has an Euler path.

Not possible without revisiting sections.





Can't draw the lines all in one path without repeats