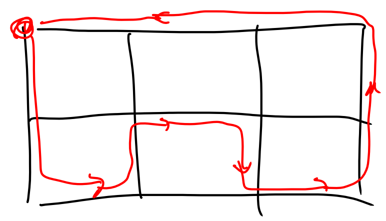
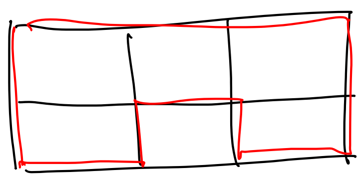
Hamilton Circuits

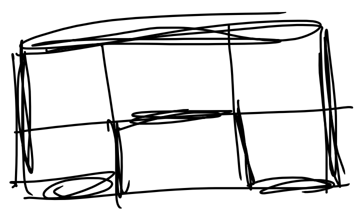
Uses every vertex, no repeats.

No cute trick to tell if Ham circs exist.



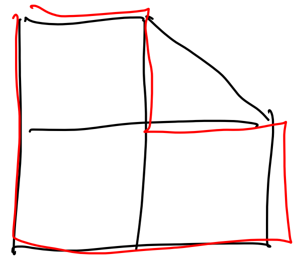
this is a Ham. circ.





How to find Ham. circuits?

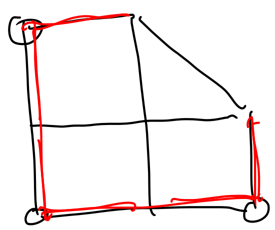
(or show there isn't one)



This is a Ham. circ.

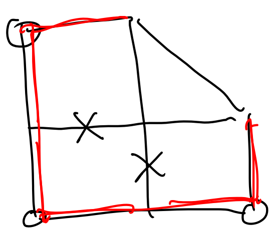
Obvious fact: Each vertex meets 2 chosen edges.

Without guessing:



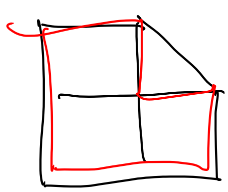
Look for degree 2 verts, those 2 edges must be used in any Ham. circ.

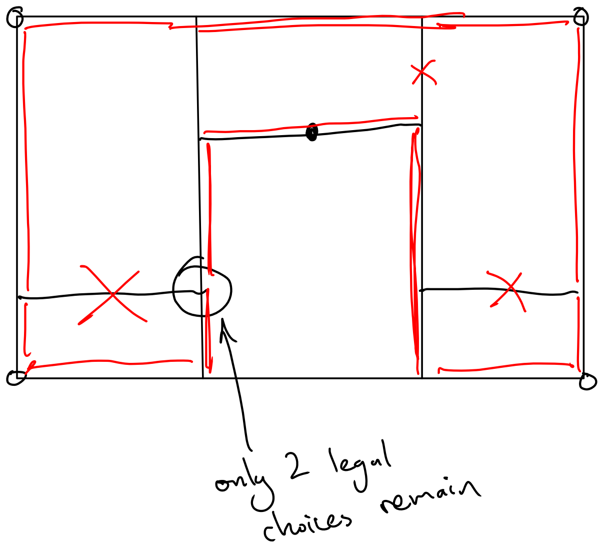
We can fill in most of it immediately.



These can't be used since they make 3 chosen edges meet.

Now in the middle only 2 edges remain, so we use both.

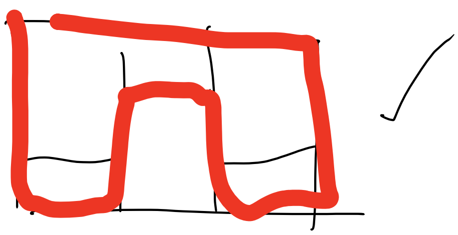


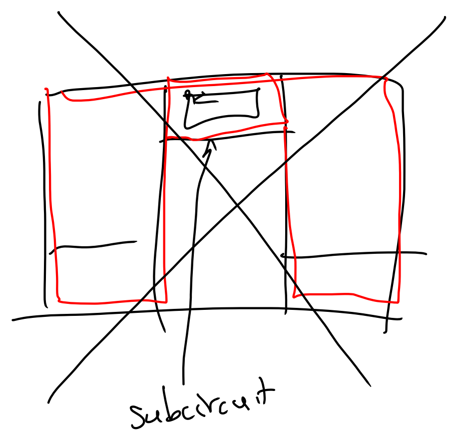


2 chosen edges per vertex

Another obvious observation:

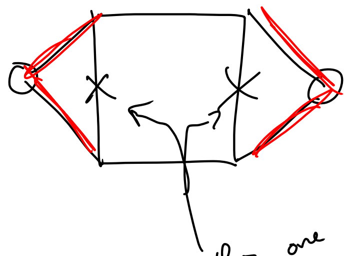
A Ham circ is one big circuit with no subcircuits





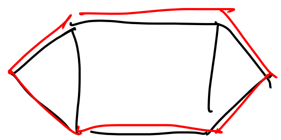
2 big ruels

- Each vert. meets 2 chosen edges
- Chosen edges never make a subcircuit.



this one would make a subcircuit.

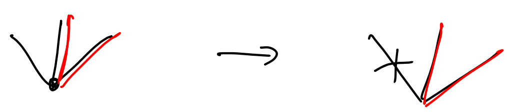
so it must be

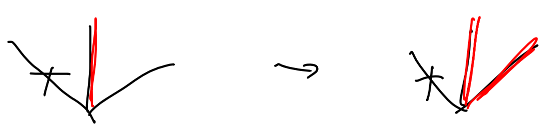


Building Ham circs by edge-choosing

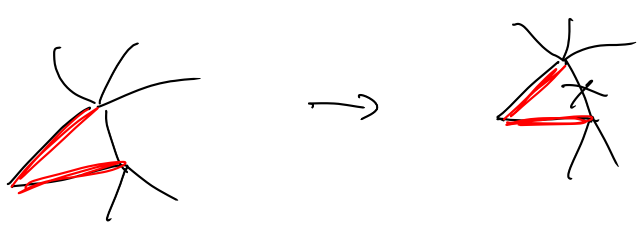
2 edges meet a vert:

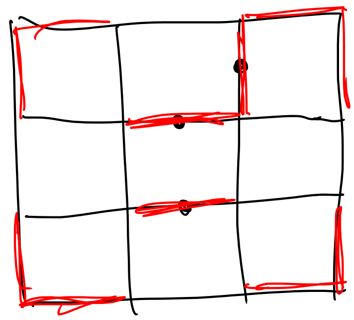
if I see 

if I see 

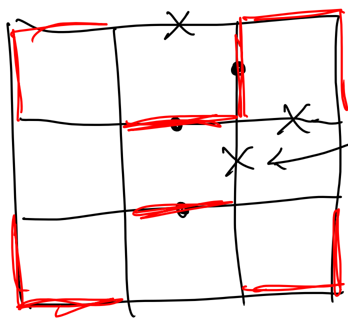
if I see 

No subcircuits



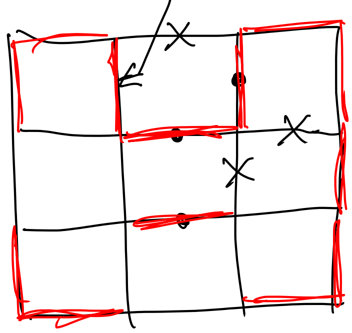


Choose edges at degree 2 verts

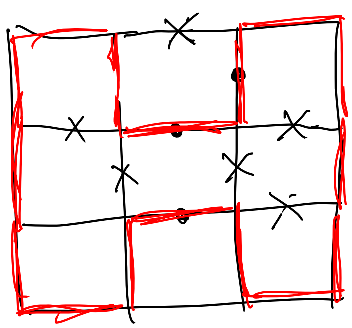


would make $\dashv$, not allowed

These would make 3-way intersections.

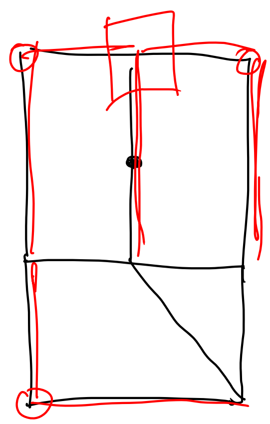


we only had one free edge, so choose it!



We can use the same tricks to show
no Ham circuit exists:

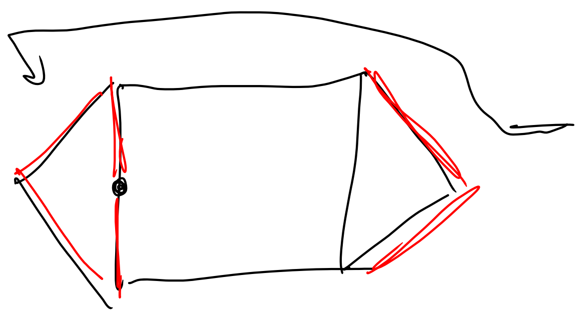
show why either 3-way intersection
or subcircuit is unavoidable.



Try to make one:

We were forced into
a violation!

So the graph has no Ham.
circuit.

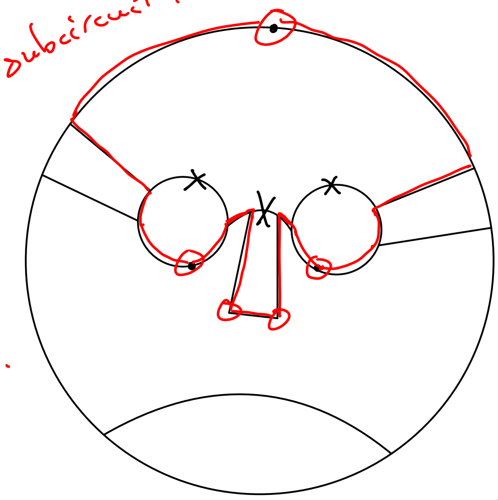


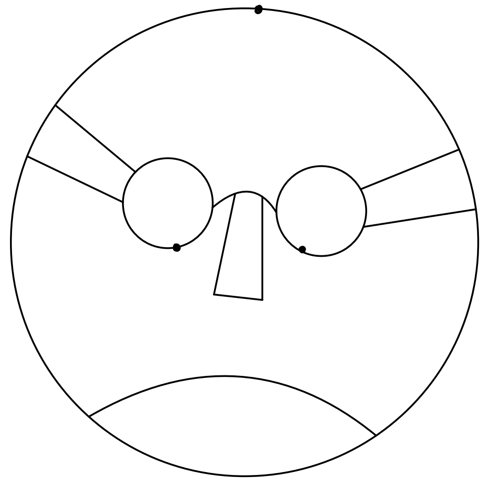
forced into a subcircuit,

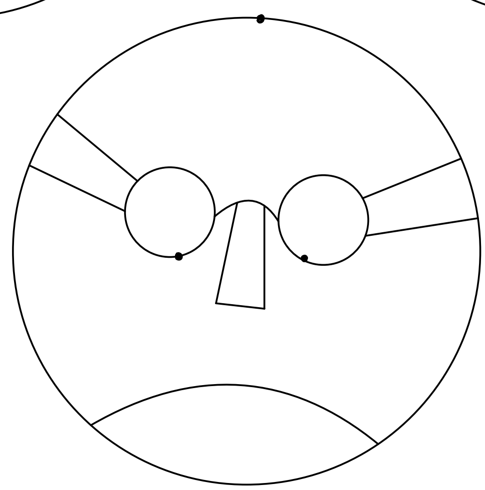
So NO Ham. circuit!

We made a subcircuit!

So no Ham circ. in this graph.

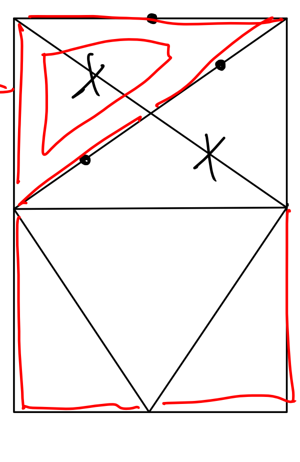


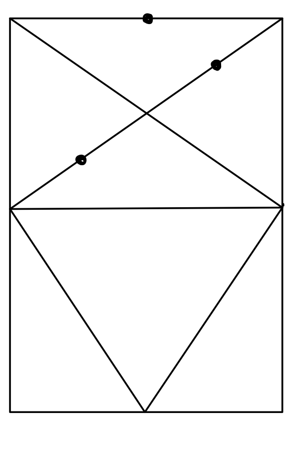


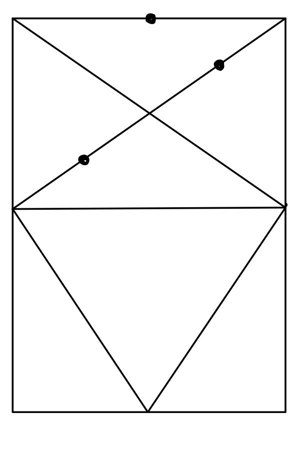


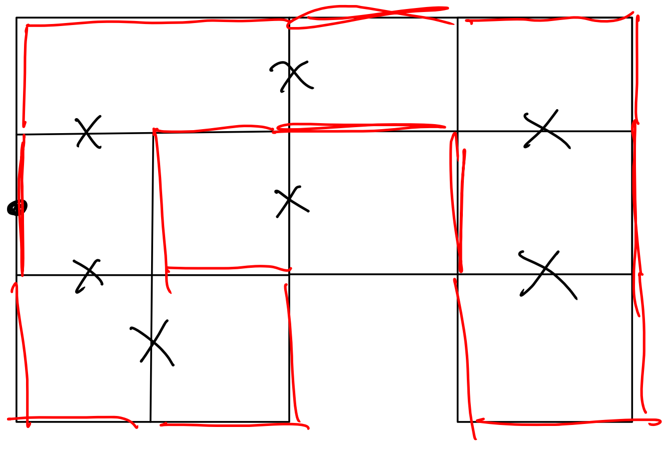
Subcircuit!

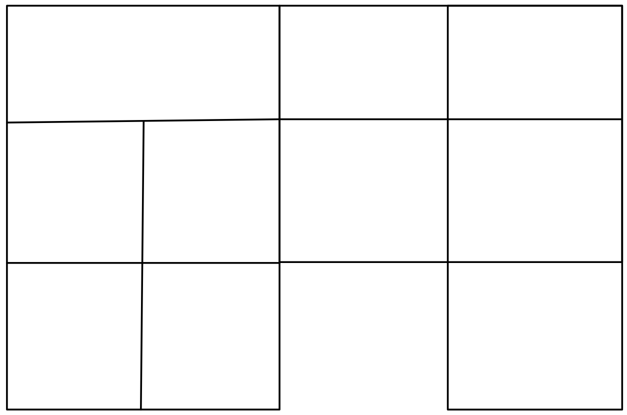
No Ham circ.

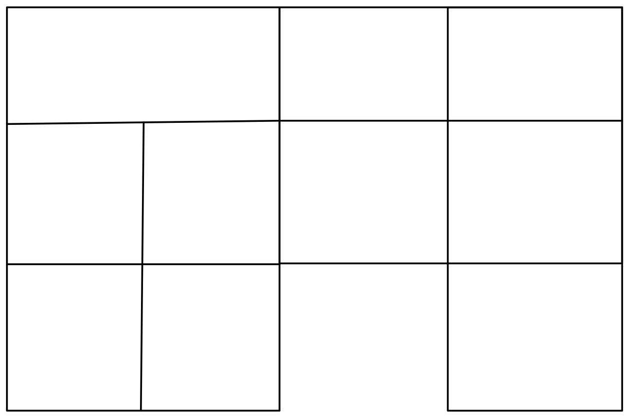






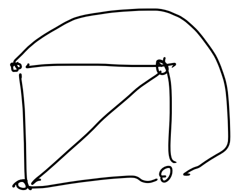


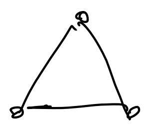




Hamilton circuits in Complete graphs

A graph is complete when any 2 verts are connected by an edge.

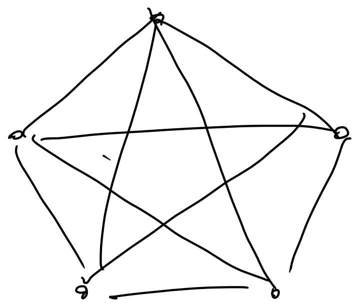


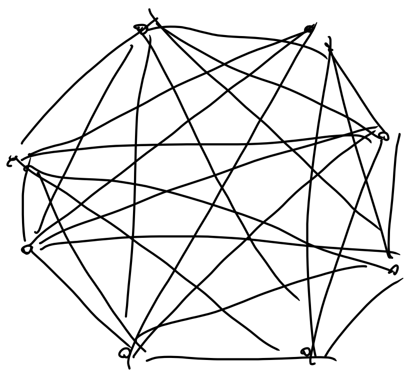


K_4 = Complete graph of 4 verts

K_3 = Complete graph of 3 verts.

K_5 =





K_8.