Math 3371

Homework #1

1.2.3a, 1.2.10,

1.3.1b, 1.3.7

<u>1.2.3a</u> False! Let $A_n = (-1/n, 1/n)$

then each A_n is infinite,

but $\bigcap_{n=1}^{\infty} A_n = \{0\}$ so the intersection is not infinite.

<u>1.2.10</u>

<u>a</u> False: let $a = b$

then "$a < b + \varepsilon \ \forall \varepsilon > 0$" is true,

but "$a < b$" is false.

<u>b</u> False: for the same reason.

(Don't read it backwards: it says:

if $a < b + \varepsilon \ \forall \varepsilon > 0$, then $a < b$.)

<u>c</u> True

<u>1.3.1</u> <u>b</u> <u>Lemma</u> Assume $s \in \mathbb{R}$ is an upper bound for a set $A \subseteq \mathbb{R}$. Then $s = \inf A$ iff $\forall \varepsilon > 0 \ \exists a \in A$ satisfying $s + \varepsilon > a$.

1.3.7 If a is an upperbound for A and $a \in A$, then $a = \sup A$.

Pf We know a is an upper bound, so we need only show it is the least upper bound.

If you like direct proof → Let b be some other upper bound, and we will show $a \le b$. Since b is an upper bound and $a \in A$, we have $b \ge a$. Shown

If you like proof by contradiction → For a contradiction, assume $b < a$ is some smaller upper bound. Since $a \in A$, b is less than some element of A which is a contradiction, because b is an upper bound.