Math 3371

Homework #2

1.4.3, 1.4.4, 1.4.8c, 1.5.2

1.4.3 For a contradiction, assume $\bigcap_{n=1}^{\infty} (0, 1/n) \neq \emptyset$,

so let $x \in \bigcap_{n=1}^{\infty} (0, 1/n)$ so $x \in (0, 1/n)$ for every n.

this means $0 < x < 1/n$ for every n, which is impossible by the Arch. Prop. (small version).

1.4.4 $T = \mathbb{Q} \cap [a,b]$, show $\sup T = b$.

Pf Since $T \subseteq [a,b]$, b is an upper bound of T.

a version using Lemma 1.3.8

we'll show $b = \sup T$ using Lemma 1.3.8. Let $\varepsilon > 0$ be given, and we'll show $\exists t \in T$ with $b - \varepsilon < t < b$.

Since $b-\varepsilon$ & b are real #s, by the density of $\mathbb{Q}$ there is some $t \in \mathbb{Q}$ with $b - \varepsilon < t < b$, and this t will be in T since it's rational.

So we showed $\exists t \in T$ s.t. $b - \varepsilon < t < b$. Shown.

a version from scratch.

We'll show it's the least upper bd by contradiction: Assume there's another upper bound $x < b$. Then by density of $\mathbb{Q}$, $\exists q \in \mathbb{Q}$ with $x < q < b$.

But $q \in T$, so this contradicts that x is an upper bound of T. Shown.

1.4.8c

Nested unbounded closed intervals with empty intersection:

$I_n = [n, \infty)$.

Then $\bigcap_{n=1}^{\infty} I_n = \emptyset$.

1.5.2

It's true NIP implies $\bigcap_{n=1}^{\infty} I_n \neq \emptyset$,

so there is some $x \in \mathbb{R}$ outside of the list.

But this doesn't make any contradiction

because x may not be in $\mathbb{Q}$. So it

doesn't make $\mathbb{Q}$ uncountable.