# Math 3371

## Homework #3

#2c, 4a, 5b, 7ab

**#2c** $\lim \frac{\sin(n^2)}{\sqrt[3]{n}} = 0$

Pf Let $\varepsilon > 0$ be given, will find $N \in \mathbb{N}$ s.t.

$n > N \Rightarrow \left| \frac{\sin(n^2)}{\sqrt[3]{n}} - 0 \right| < \varepsilon$.

$\left| \frac{\sin(n^2)}{\sqrt[3]{n}} \right| \leq \frac{1}{\sqrt[3]{n}}$, want $\frac{1}{\sqrt[3]{n}} < \varepsilon$

$\frac{1}{n} < \varepsilon^3$

$n > \frac{1}{\varepsilon^3}$

Let $N > \frac{1}{\varepsilon^3}$. Then if $n > N$ we have:

$$\left| \frac{\sin(n^2)}{\sqrt[3]{n}} - 0 \right| = \left| \frac{\sin(n^2)}{\sqrt[3]{n}} \right| \leq \frac{1}{\sqrt[3]{n}} < \frac{1}{\sqrt[3]{N}} < \frac{1}{\sqrt[3]{1/\varepsilon^3}} = \varepsilon$$

Shown.

**#4a** This is possible:

$(1, 0, 1, 0, \ldots)$ has an infinite # of 1s, but does not converge to 1.

#5b $\left[\left[\frac{12+4n}{3n}\right]\right] \to 1$

Pf let $\varepsilon > 0$ be given, we'll find $N \in \mathbb{N}$ s.t.

$$n > N \implies \left| \left[\left[\frac{12+4n}{3n}\right]\right] - 1 \right| < \varepsilon.$$

$$\frac{12+4n}{3n} = \frac{12}{3n} + \frac{4n}{3n} = \frac{4}{n} + \frac{4}{3}$$

when $n > 6$, this is all less than 2 and more than 1.

Let $N = 6$. Then if $n > N$, we have

$$\frac{12+4n}{3n} = \frac{4}{n} + \frac{4}{3} < \frac{4}{6} + \frac{4}{3} = 2$$

and $\frac{12+4n}{3n} = \frac{4}{n} + 4/3 > 4/3$

so $\frac{12+4n}{3n}$ is between $4/3$ & $2$, so

$$\left[\left[\frac{12+4n}{3n}\right]\right] = 1,$$

so $\left| \left[\left[\frac{12+4n}{3n}\right]\right] - 1 \right| = 0 < \varepsilon$ as desired.

#7 <u>a</u> $(-1)^n$ is frequently, but not eventually in $\{1\}$.

<u>b</u> eventually $\Rightarrow$ frequently, but not the converse.