Math 3371

HW #4

2.3 #1a, 2a, 4b, 7ac

2.3 #1a If $(x_n) \to 0$ then $(\sqrt{x_n}) \to 0$

Pf Let $\varepsilon > 0$ be given, we'll show $\exists N$ s.t. $n > N \Rightarrow |\sqrt{x_n} - 0| < \varepsilon$

$|\sqrt{x_n} - 0| = \sqrt{x_n} < \sqrt{\varepsilon^2} = \varepsilon$

make $x_n < \varepsilon^2$

Choose N so big so that $n > N \Rightarrow |x_n - 0| < \varepsilon^2$,

i.e. $|x_n| < \varepsilon^2$

Then if $n > N$, we have:

$|\sqrt{x_n} - 0| = \sqrt{x_n} < \sqrt{\varepsilon^2} = \varepsilon$ as desired.

2.3 #2a If $(x_n) \to 2$ then $\left(\frac{2x_n - 1}{3}\right) \to 1$

Pf Let $\varepsilon > 0$ be given, we'll find N s.t.

$$n > N \Rightarrow \left| \frac{2x_n - 1}{3} - 1 \right| < \varepsilon$$

$$\left[\left| \frac{2x_n - 1}{3} - 1 \right| = \left| \frac{2x_n - 4}{3} \right| = \frac{2}{3} |x_n - 2| \right.$$

Choose N so large that $n > N \Rightarrow |x_n - 2| < 3/2\, \varepsilon$.

then if $n > N$, we have:

$$\left| \frac{2x_n - 1}{3} - 1 \right| = \frac{2}{3} |x_n - 2| < \frac{2}{3} \cdot \frac{3}{2} \varepsilon = \varepsilon$$

Shown

2.3 #4b

$$\lim \frac{(a_n + 2)^2 - 4}{a_n} = \lim \frac{a_n^2 + 4a_n + \cancel{4 - 4}}{a_n} = \lim \frac{a_n^2 + 4a_n}{a_n}$$

$$= \lim (a_n + 4) = (\lim a_n) + (\lim 4)$$

$$= 0 + 4 = 4$$

2.3 # 7a

$x_n = (1, 2, 3, \ldots)$

$y_n = (-1, -2, -3, \ldots)$

Then x_n & y_n diverge, but $x_n + y_n = (0, 0, 0, \ldots)$ converges.

#7b Impossible!

if (x_n) converges and $(x_n + y_n)$ converges,

then $(x_n + y_n) - (x_n)$ converges,

so (y_n) converges.