Math 3371 HW # 7

Section 3.3 # 11 b

3.4 # 7

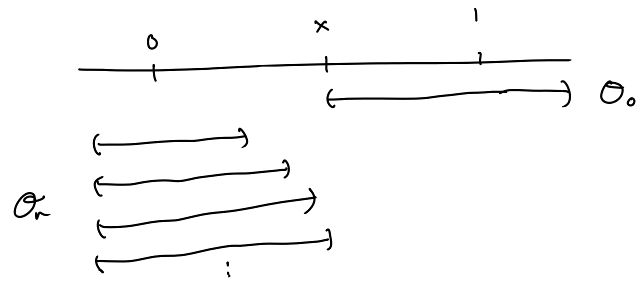
4.2 # 5a, c

3.3 # 11b $\quad \mathbb{Q} \cap [0,1]$

Choose some irrational $x \in [0,1]$ (like $\frac{1}{\pi}$, etc)

let $\mathcal{O}_0 = (x, 2)$

$\mathcal{O}_n = (-1, x - \frac{1}{n})$



This has no finite subcover.

3.4 # 7 a) Take $x, y \in \mathbb{Q}$, assume $x < y$

then choose some irrational z with $x < z < y$,

let $A = (-\infty, z) \cap \mathbb{Q}$,

$B = (z, \infty) \cap \mathbb{Q}$.

Then A & B are separated since $\bar{A} = (-\infty, z]$ and $B = (z, \infty) \cap \mathbb{Q}$

so $\bar{A} \cap B = \emptyset$

and similarly $A \cap \bar{B} = \emptyset$.

b Yes - same idea: given $x, y \notin \mathbb{Q}$, choose $z \in \mathbb{Q}$ with $x < z < y$,

then let $A = (-\infty, z) - \mathbb{Q}$

$B = (z, \infty) - \mathbb{Q}$

4.2 #5a $\lim_{x \to 2} 3x+4 = 10$

Pf Let $\varepsilon > 0$ be given, we'll find $\delta > 0$ s.t.

$$0 < |x-2| < \delta \implies |3x+4-10| < \varepsilon$$

$$|3x+4-10| = |3x-6| = 3|x-2|$$

Let $\delta = \varepsilon/3$, then if $|x-2| < \delta$ we have

$$|3x+4-10| = 3|x-2| < 3 \cdot \varepsilon/3 = \varepsilon$$ as desired.

#5c $\lim_{x \to 2} x^2+x-1 = 5$

Pf Let $\varepsilon > 0$ be given, we'll find $\delta > 0$ s.t.

$$0 < |x-2| < \delta \implies |x^2+x-1-5| < \varepsilon$$

$$|x^2+x-1-5| = |x^2+x-6| = |x+3||x-2|$$

let $\delta < 1$, then $x < 3$ so $|x+3| < 6$

so $\implies \; < 6|x-2|$

Let $\delta = \min(1, \varepsilon/6)$. Then if $|x-2| < \delta$ we have

$$|x^2+x-1-5| = |x+3||x-2| < 6|x-2| < 6 \cdot \varepsilon/6 = \varepsilon$$ as desired.