Math 3371 HW #8

Section 4.2 # 8a, 9

Section 4.3 # 7a, 8b

4.2 #8a $\lim_{x \to 2} \frac{|x-2|}{x-2}$ does not exist.

Let $x_n = 2 + 1/n$ Then we'll show $f(x_n)$ & $f(y_n)$
$y_n = 2 - 1/n$ have different limits.

$$f(x_n) = \frac{|2 + 1/n - 2|}{2 + 1/n - 2} = \frac{1/n}{1/n} = 1 \to 1$$

$$f(y_n) = \frac{|2 - 1/n - 2|}{2 - 1/n - 2} = \frac{1/n}{-1/n} = -1 \to -1$$

4.2 #9 $\lim_{x \to 0} \frac{1}{x^2} = \infty$

Let $M > 0$ be given, we'll find $\delta > 0$ s.t.

$$0 < |x - 0| < \delta \Rightarrow \left|\frac{1}{x^2}\right| > M.$$

Simplify $\left|\frac{1}{x^2}\right| = \frac{1}{|x|^2}$ so to get $\frac{1}{|x|^2} > M$, we need
$|x| < \frac{1}{\sqrt{M}}$

Let $\delta = \frac{1}{\sqrt{M}}$. Then if $|x| < \delta$, we have

$$\left|\frac{1}{x^2}\right| = \frac{1}{|x|^2} > \frac{1}{\delta^2} = \frac{1}{(1/\sqrt{M})^2} = M \text{ as desired.}$$

4.3 #7a

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q}. \end{cases}$$

Take any $c \in \mathbb{R}$, we'll show f is not continuous at c

let x_n be a seq. of rationals with $x_n \to c$,
y_n - - - - - irrationals with $y_n \to c$.

then $f(x_n) = 1 \to 1$
$f(y_n) = 0 \to 0$

so x_n & y_n have the same limits but $f(x_n)$ & $f(y_n)$ have different limits,
so f is not continuous.

4.3 #8b

Let $g(r) = 0 \quad \forall r \in \mathbb{Q}$, let $x \in \mathbb{R}$,
we'll show $g(x) = 0$.

Choose some seq. of rationals $x_n \to x$.
Then since g is continuous, $\lim g(x_n) = g(x)$.
but $g(x_n) = 0 \ \forall n$ since $x_n \in \mathbb{Q}$, so

$$g(x) = \lim g(x_n) = 0.$$

Shaun